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Abstract

The valuation of any mixed-use development site must perforce grapple with the allocation of the site to the permitted uses as market valuation is premised upon highest and best (HBU) use. The optimization problem involved in such cases prejudices the sole use of traditional valuation methods which cannot deal with the inherent allocation problem. Furthermore the synergism among the permissible uses introduces more complexities to the problem. Thus, the assumption of a linear relationship among the permissible uses may not suffice; neither is it intuitively appealing – A nonlinear model is used to resolve the optimization problem involved in the exercise before valuing the site via the Residual method. While the nonlinear model allocates the site to all six permissible uses, the linear model allocates the site to five of the uses. Furthermore, the nonlinear model results in a gross development value and a site value that are 22.04% and 39.81% respectively higher than the linear model.

Key words: Mixed-use development, synergy, management science, valuation, residual method.

1. Introduction

The quest for sustainable real estate, as a result of the sustainability movement, is providing more impetus for mixed-use developments. Although there is ample literature on the concept of highest and best use (e.g. Graaskamp 1970, North 1981, Grissom 1983, Lennhoff 1995, Sarazen 1995, Wilson 1995 & 1996 and Finch 1996) the extant literature mainly focuses on various aspects of highest and best among competing alternate uses (see Addae-Dapaah). Apart from Gau and Kohlhepp (1980), Winokur et al. (1981), Peiser and Andrus (1983), Mouchly and Peiser (1993) and Addae-Dapaah (2005), the extant literature does not deal with the complex issue of ascertaining the best mix of uses (as may pertain to a mixed-use development site) that maximizes return, and thus, constitutes the highest and best “use” (HBU).

Gau and Kohlhepp (1980) use linear programming to generate a development/construction schedule which maximizes the profitability of a real estate project. Winokur et al. (1981) employ
linear programming to propose a land development strategic plan that maximizes the expected present value of the future cash flows of a real estate development project. Furthermore, Peiser and Andrus (1983) utilize integer programming to schedule office construction while Mouchly and Peiser (1993) use linear programming to deal with optimal land use plan. However, none of these studies relates to valuation of the site. Given that some of these mixed-use development sites are aimed at giving the prospective developer maximum flexibility in choosing the “best” permissible single, or mix of, use(s), a valuer must ideally ascertain the best mix of uses that promotes both efficient land utilization and maximization of profits to underpin the valuation of the site.

Mixed-use development sites are normally offered for sale in areas where there are no market value/rental data for development land. Therefore valuers appropriately rely on the Residual method and/or cash flow analysis for valuing such sites. Addae-Dapaah (2005) finds that the usefulness of the Residual method for valuing a mixed-use development site could be severely compromised as the method, and even the much trumpeted cash flow method, per se cannot resolve the inherent optimization problem in the valuation of a mixed-use development site. Furthermore, Addae-Dapaah (2005) observes that studies that use the real option model to explain the valuation of land and factors affecting investment decisions, such as Titman (1985), Capozza and Hesley (1990), Williams (1991), Grenadier (1995) and Paxson (2005) deal with the option in relation to a single use rather than a mix of uses. Moreover, all the studies that utilize artificial neural network (ANN) for real estate appraisal (e.g. Tsukuda and Baba, 1990; Do and Grudnitski, 1992; Tay and Ho, 1992; Allen and Zumult, 1994; Worzala et al., 1995; Bagnoli et al., 1998; Nguyeh and Cripps, 2001; Peterson and Flanagan, 2009) relate to single use developments or sites that are already developed rather than sites that can be put to a mix of uses.

Geltner et al. (1996) deal with the choice between two underlying assets rather than a choice among several independent alternate uses or mix/different mixes of uses, whichever maximizes return. Similarly, Capozza and Li (1994), inter alia, deal with the conversion of vacant land to urban use vis-à-vis price of the land as a function of option premium. The extant literature on fuzzy logic and ANN vis-à-vis property valuation, in addition to providing us with conflicting views on the relative performance of both soft computing approaches and the hedonic model,
deals with the nonlinearity between the value of a home and some of the factors affecting property value as well as the predictive power of ANN (see for example, Grether and Mieszkowski, 1974; Do and Grudnitski, 1993; Goodman and Thibodeau, 1995; Liu et al., 2006; Lam et al., 2008; Peterson and Flanagan, 2009). The extant literature on soft computing does not deal with valuation of mixed-use properties which involve optimizing the allowable uses. However, with the development of the multi-layered perception, soft computing is capable of handling optimization problems (Lee and Paik, 2006) and thus, deal with the problem inherent in the valuation of a mixed-use development site provided there is sufficient market data for the training involved in ANN.

Addae-Dapaah (2005) appears to be the only researcher who has used the linear programming (LP) model to find the mix of uses that maximizes return for a mixed-use development site. However, Addae-Dapaah (2005) assumes that the values for the permitted land uses encapsulate the synergy among the related uses to justify the LP model. While this is plausible, it must be noted that some mixed-use developments occur in areas where it is almost impossible to find analogs that encapsulate the synergy that a proposed mixed-use development will generate. For example, the “white site”, a novel form of mixed-use development sites, which Addae-Dapaah (2005) uses for his study, was reclaimed from the sea. There were no acceptable analogs (when “white sites” were first offered for sale) to justify the assumption that the values capitalized the synergism of the related land uses. This implies that Addae-Dapaah’s (2005) study that is premised on the assumption of linear relationships among the permissible land uses and their values could be simplistic – A nonlinear model that accounts for the synergy among the alternate competing uses may be more appropriate.

Thus, a gap exists in the literature on the mix of uses (allocation problem) that maximizes return (which accounts for the synergy among the various land uses) for a mixed-use development site to constitute the HBU. This gap may be filled by utilizing soft computing – a collection of methodologies that work synergistically to provide information processing capabilities for handling real-life situations (Sethuraman, 2006). According to Jain et al. (1996: ), the Hopfield network can be used to find the optimal (or suboptimal) solution if a combinatorial optimization problem, such as the classical travelling salesman problem, can be formulated in terms of
minimizing network energy function. However, ANN may be unsuitable for the current task as it requires market data (which are not available) for training – Mixed-use development sites often do not have analogs to provide the required data for training ANN. Furthermore, Sethuraman (2006:203) states: “Currently, fuzzy logic (Wang, 1997), artificial neural networks (Hassoun, 1995; Mehrotra et al., 1997) and genetic algorithms (Everitt and Dunn, 2001) are three main components of soft computing. ANN is suitable for building architectures for adaptive learning, and GA can be used for search and optimization. Fuzzy logic provides methods for dealing with imprecision and uncertainty. The analytical value of each one of these tools depends on the application”. Given the inability of fuzzy logic and ANN to explain their behaviour (Sethuraman, 2006) and that many design parameters still have to be determined by the trial-and-error method (Jain et al., 1996), LP and nonlinear programming (NLP), with their shadow pricing, Lagrange multipliers, active and inactive constraints etc., appear to be more suitable than fuzzy logic and ANN in the valuation of mixed-use development site. Our bias is tilted towards LP/NLP. Thus, this study attempts to fill the gap (and therefore contributes to existing literature) by utilizing LP and NLP to model the HBU for a mixed-use development site to show that the NLP model provides better results which, combined with Residual or any traditional method of valuation, will produce a more scientifically based and supportable estimate of value for a mixed-use development site.

The study is organized as follows. A mixed-use development site and the attendant valuation problems are briefly discussed. This is followed by a theoretical development of the problem in the context of management science. The sections that follow cover methodology and data sourcing and management, empirical modelling of the HBU, presentation and interpretation of the results, and a valuation of a mixed-use development site in Singapore. It is concluded that the use of linear programming model to value a mixed-use development site could be problematic as the nonlinear model (which accounts for synergy among the uses) result in a gross development value and site value that are 22.04% and 39.81% higher than those of the linear model.

2. Mixed-use Development Site and the HBU Puzzle in Valuation
The HBU puzzle involved in the valuation of a mixed-use development site, and the similarities and differences between a traditional mixed-use and “white site” have been articulated by
Addae-Dapaah (2005). The peculiar feature of a “white site” is the maximum flexibility offered to the owner to put the site to any single use or mix of the permissible uses, and change the use/mix of uses whenever he deems fit without recourse to the planning authority.

Therefore the owner of the site has the problem of allocating scarce resources (e.g. land, money, etc.) among alternate competing uses at different times to maximize his profit subject to financial, technological and institutional constraints that limit his prerogatives – the owner of the site has “constrained” options. Thus, option pricing could lend itself to the valuation of such sites albeit the Residual valuation and discounted cash flow valuation being the norm in Singapore. Addae-Dapaah (2005) argues that both the Samuelson-McKean (1965) method for valuing vacant land and the Residual method of valuation are premised on the HBU of the land. Furthermore, it is argued that the two models presume, rather than resolve, the HBU problem which the valuation of “white sites” (and any mixed-use development site) poses (Addae-Dapaah 2005). Similarly, cash-flow-based valuation is based on HBU which is presumed. The computations involved in ascertaining the highest and best use or mix of uses (HBMU) that will maximize utility (i.e. land value) given the institutional, economic, physical and technological constraints of a particular “white site” that permits residential, retail, restaurant, office, hotel and Cineplex developments or any combination thereof could be numerous, while the calculations involved in a trial-and-error method could be tedious and quite inefficient in terms of time and cost.

3. Relevance of Management Science

The brief discussion on “white site” and valuation reveals that optimization (i.e. combining the permissible land uses to produce the highest return) is pivotal to the valuation of a mixed-use development site. The complexity of the problem is compounded by the fact that the land value attributable to each use is inextricably intertwined with the other permissible uses because of the synergy that exists among the uses. Furthermore, the actions of whoever owns the site are constrained by political, social, economic, planning etc. obligations. Thus, apart from physical (plot size, and land use capacity and intensity), resources and planning limitations on the site’s development, a developer has to grapple with the following two challenges if he is to maximize returns (see Addae-Dapaah, 2005):
a) The permissible economic use(s) to be developed given that some, or all, of these uses could be functionally complementary or incongruous to have synergistic effect(s); and
b) Marginal revenue and opportunity cost attributable to each permissible economic use. The marginal revenue for each use could be affected by the marginal revenue(s) of other use(s) because of the synergism among the permissible uses. This makes the combination of the uses very challenging as the synergism among the permissible uses can have either positive or negative impact on the associated marginal revenues and opportunity costs.

In addition, the prevalence of institutional, financial, physical, etc. constraints calls for complex trade-offs among the variables associated with a mixed-use development site. As noted by Winokur et al. (1981:50), “Management Science techniques are particularly well suited to handling such complex trade-offs systematically, in a manner which presents and evaluates alternatives for management”. Since the valuation exercise primarily centres on the allocation of a scarce resource (i.e. land) among alternate competing uses and different mixes thereof, management science (in particular LP and NLP) would appear to be the best analytical tool before utilizing the appropriate valuation method(s) to estimate the value of the site.

4.1 Methodology
The LP and NLP models are employed to resolve the allocation problem in the case before applying the Residual method to value the site. The LP and NLP models can be expressed as the minimization/maximization of a (non)linear function subject to (non)linear inequality and/or equality constraints (Rardin 1998; Carter and Price 2001; Avriel, 1976). Both models may generally have three fundamental variables: a finite number of real variables (land use in our case), a finite number of constraints (e.g. resource availability and institutional controls) which the variables must satisfy, and a function of the variables which must be maximized/minimized (i.e. land value/cost in our case). In mathematical terms, we are finding specific values \( (\bar{x}_1, \ldots, \bar{x}_n) \), if they exist, of the variables \( (x_1, \ldots, x_n) \) that will satisfy:

Maximize/minimize

\[
\theta(x_1, \ldots, x_n) \tag{1}
\]
subject to:

\[ g_i(x_1, \ldots, x_n) \leq 0; \quad i = 1, \ldots, m \]  \hspace{1cm} (2)

\[ h_j(x_1, \ldots, x_n) = 0; \quad j = 1, \ldots, k \]  \hspace{1cm} (3)

where \( \theta, g_i \) and \( h_j \) are numerical functions of the variables \( x_1, \ldots, x_n \), which represent the permissible economic land uses (e.g. condominium, retail, etc.). \( \theta \) depicts the marginal revenue (i.e. value per m\(^2\)) associated with the corresponding permissible economic land uses. Alternatively, \( \theta \) may be construed as the opportunity cost for not putting the site to the relative uses. If the functions \( \theta, g_i \) and \( h_j \) are linear in the variables \( x_1, \ldots, x_n \), then the optimization problem (1)-(3) is known as a linear program; otherwise, it is classified as a nonlinear program (Avriel, 1976).

4.1.1 Differences between LP and NLP

LP and NLP problems differ in model specification and level of difficulties in solving problems. Although both models generally have three fundamental variables as explained above, one main difference between LP and NLP is that a NLP can have a nonlinear objective function and/or one or more nonlinear constraints (Ragstale, 2001). Secondly, LP problems can be solved with relative ease compared to NLP problems because:

a. LP problems lend themselves to “corner” or extreme point solutions. Given any bounded LP problem, it is certain that one “corner” or extreme point of the feasible solution is optimal. (Thus, in principle, it is only necessary to search over a finite number of feasible solutions – extreme points – to solve an LP problem, but one should note that such an exhaustive search is computationally impractical as there is exponential number of extreme points.) The Simplex method, originally proposed by G. Dantzig, can efficiently solve LPs with hundreds of variables and constraints. It takes advantage of the extreme-point property of the optimal solution by traversing from extreme point to extreme point to improve on the optimality of the solution.

b. An extreme point, which provides an objective function value that cannot be improved by any movement away from it, must and is the optimal solution to the problem. In other words, any locally optimal solution (i.e. a solution that is better than any other feasible solution in its immediate, or local, vicinity) must also be the optimal over the whole feasible region (i.e. global optimal).
In contrast, NLPs do not generally have the above beneficial properties of LPs. NLPs do not generally have extreme point solutions. This implies that there is no analogue of the Simplex method for solving a NLP. A variety of sophisticated methods relying on local linear/quadratic function approximations to iteratively solve a NLP problem are currently available as a result of the great advances in the optimization community in the past three decades. However, these methods would usually take more computing time to solve a NLP compared to the Simplex method on a LP with the same number of variables and constraints. Apart from computational tractability, the challenge in solving a NLP is compounded by the fact that NLPs could have local optima that might not be overall optimal. Thus, one should ideally find all the local optima in NLPs to be able to ascertain the overall optimum. This sharply contrasts with LPs which are solved once a local optimum is found.

Another problem that can arise in NLP is the possibility for the feasible region to have two or more entirely disconnected sets of points. Thus, any “generalized reduced gradient” or “path following algorithm (as pertains in the simplex method for solving LPs) may not enable us to find the overall optimal solution as the algorithm cannot cross from one segment of a disconnected feasible region to another. The problem is essentially the same as that which can emanate from the existence of different local optima in a fully connected feasible region – The local optimum solution at which a “path-following” algorithm terminates depends almost entirely on the initial starting point (see Simmons, 1975; Bertsekas, 1995; Ragsdale, 2001).

These difficulties in NLPs are dealt with systematically based on duality theory, and Lagrange Multipliers and Kuhn-Tucker theory. Lagrange Multipliers deal with unconstrained and equality constrained problems while Kuhn-Tucker optimality theory (Kuhn and Tucker, 1951) is the theoretical underpin for virtually all the solution methods of NLP (Simmons, 1975; Bazarra & Shetty, 1990).

4.1.2 Kuhn Tucker Conditions

To give the reader an idea of the Kuhn Tucker optimality theory, we consider the following NLP without equality constraints:

\[ \text{Max } f(X) \quad X = (x_1 \ldots x_n) \]
\[ \text{s.t. } g_i(X) \leq 0, \quad i = 1, 2, \ldots, m \]
\[ X \geq 0, \]
where the functions \( f(X), g_i(X) \) are assumed to be continuously differentiable in the feasible region. The nonlinear HBU model which we shall study in Section 4.2 has exactly the above form, where \( f(X) \) is a nonlinear function but \( g_i(X) \) are all linear functions. The most general form of NLP stated in (1)—(3) is beyond the tenet of this paper, and we refer interested readers to see for example: Bazaraa and Shetty, 1990; Ragsdale, 2001; Kasana and Kumar, 2004 for detailed explanation.

The necessary conditions for \( X^* \) to be a local optimal solution of the problem are:

\[ \frac{\partial L(X^*, \lambda^*)}{\partial x_j} \leq 0; \quad x_j^* \frac{\partial L(X^*, \lambda^*)}{\partial x_j} = 0, \quad j = 1, 2, \ldots, n \]

\[ \frac{\partial L(X^*, \lambda^*)}{\partial \lambda_i} = g_i(X^*) \leq 0; \quad \lambda_i^* \frac{\partial L(X^*, \lambda^*)}{\partial \lambda_i} = 0, \quad i = 1, 2, \ldots, m \]

\[ x_j^* \geq 0, \lambda_i^* \leq 0 \]

where \( L \equiv L(X, \lambda) \) is the Lagrange function:

\[ f(X) + \sum_{i=1}^{m} \lambda_i g_i(X) \]

Equations (4)-(6) are the Kuhn-Tucker (K-T) necessary conditions for optimality. Except for rare pathological cases, it means that any local optimal solution \( X^* \) of the NLP can be found among the solutions satisfying the K-T necessary conditions. There are sufficient conditions (known as K-T second order sufficient conditions, see Bazaraa and Shetty, 1990; Kasana and Kumar, 2004) to determine which K-T solutions are local maxima which we shall not present here for brevity.

As the name suggests, a local maximizer \( X^* \) is a point in the feasible region of the NLP which gives the largest objective function value among points in a local vicinity of \( X^* \). If \( X^* \) also gives the largest objective function value among all the feasible points of the NLP, it is also called a global maximizer. As stated in Simmons (1975:201), if the functions \( -f(X), g_i(X) \) are all convex functions in the feasible region, then any local maximizer is also a global maximizer.

### 4.2 Data Sourcing and Management

Data for modelling the LP/NLP problem were mainly obtained from the tender document for the white site used as a case study for the paper, Real Estate Information System (REALIS), Rider
Hunt and Bailey (RHB – a cost consulting firm in Singapore), and property consulting firms in Singapore. REALIS is an URA database, which records open market sale prices obtained from caveats lodged with Singapore Land Authority (i.e. Registrar of Title Deeds). The data are updated fortnightly on the first and sixteenth of every month. REALIS is the most reliable public database that is used by property market researchers in Singapore. The relevant inputs for modelling the allocation problem [equations (1)-(3)] are the cost and expected values per unit of the permissible land uses (assuming the land is fully developed) as well as the impact of the synergy among these land uses on the corresponding land values. The fact that the price of land in Singapore is a function of the plot ratio (i.e. building-to-land ratio) vis-à-vis a plot ratio of 6.0 as implied by the land area of 26,667.9 m² and a maximum gross floor area (GFA) of 160,016 m², suggests that a high-rise development, as opposed to low-density development of detached houses, etc. is more suited to the site. Thus, the most suitable residential development (which conforms to, or balances existing developments in the surrounding areas) that is used for modelling the problem is condominium. In addition, the planning guidelines for the site permit commercial, entertainment and hotel uses. Moreover, information from REALIS and the industry reveals that the mean value (reflecting the long term occupancy and absorption rates, etc.) and cost per m² of the permissible land uses are as follows:

<table>
<thead>
<tr>
<th>Land Use</th>
<th>Value per m²</th>
<th>Cost per m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condominium (x₁)</td>
<td>r₁ = S$12,200</td>
<td>c₁ = S$4,500</td>
</tr>
<tr>
<td>Retail (x₂)</td>
<td>r₂ = S$18,840</td>
<td>c₂ = S$4,350</td>
</tr>
<tr>
<td>Office (x₃)</td>
<td>r₃ = S$10,760</td>
<td>c₃ = S$3,950</td>
</tr>
<tr>
<td>Hotel (x₄)</td>
<td>r₄ = S$13,750</td>
<td>c₄ = S$5,650</td>
</tr>
<tr>
<td>Restaurant (x₅)</td>
<td>r₅ = S$13,078</td>
<td>c₅ = S$4,350</td>
</tr>
<tr>
<td>Cineplex (x₆)</td>
<td>r₆ = S$15,850</td>
<td>c₆ = S$3,000</td>
</tr>
</tbody>
</table>

The cost per m² (obtained from RHB and the industry) includes professional fees and financing cost but excludes land cost. Similarly, the values per m² are based on sale prices of comparable properties extracted from REALIS, and from the industry.

The residential, retail and office developments are each divided into two categories as shown in the definition of the decision variables (land uses) under the sub-section 4.2.1 “Empirical
Modelling of the Problem”. The figures in Table 1, which are extracted from a study by Teo (2006), in which 2000 sale transactions were examined in an attempt to quantify the impact of mixed-use developments on the value of the various uses, and data from property consultancy firms in Singapore, depict the impact of the synergism among the decision variables (land uses) on the values of the respective decision variables. For example, $x_{1a}$ and $x_{1b}$ (residential developments) increase the values of retail and office developments by 25% and 5% respectively. Similarly $x_{2a}$ and $x_{2b}$ reduce the values of $x_{1a}$ and $x_{3a}$ by 15% and 10% respectively and increases the values of both $x_{1b}$ and $x_{3b}$ by 20%.

Table 1

In addition to the above, information from the industry indicates that retail development requires a minimum GFA of 5,000m$^2$ in a mixed-use development to make it viable. Similarly, given the configuration of comparable Cineplex in Singapore, Cineplex in the proposed development should occupy at least a floor area of 1,000 m$^2$ (Equation (13)), and we assume the maximum floor area of 3,000 m$^2$ (Equation (14)). Other relevant data for modelling the problem are obtained from a white site in Singapore. The relevant features of the planning guidelines are as follows:

Land Use: White site – commercial/residential/hotel uses.

Gross Floor Area: The maximum permissible GFA is 160,016 m$^2$ of which at least 40% and 15% must be for office and hotel uses respectively. The remainder of the maximum permissible GFA (45%) may be used for any one or more of the following uses – commercial (e.g. office, retail and entertainment), hotel, residential.

In LP and NLP language, the minimum and maximum GFA figures are resource constraints. Furthermore, restaurants in shopping malls in Singapore are classified as retail – A portion of the retail area is used for restaurants. A shopping complex in Singapore has been allowed to use a maximum of 24% of the retail area for food and beverages (F&B). To encourage retail mix, we assume that at least 20% of the retail space must be used for F&B (Equation (15)).
In view of all these information, the total cost of a mixed-use development, which takes account of the mandatory minimum GFA requirements and assuming that the remaining GFA is equally allocated among the permissible uses is “S$719,259,550”² (see Addae-Dapaah 2005). This basis of calculating the total cost of the development is necessary to provide an initial basis for analysis as the optimal allocation of the GFA to the various uses is not known yet. Furthermore, total cost based on the mandatory minimum GFA requirements and allocating the remaining GFA to the most profitable permissible use (retail) assuming the market can absorb it (S$701,670,160), is lower than the former. Given that the latter figure does not fully account for all the synergism among the permissible uses while setting a lower bound on funds required for the project, the former figure which is based on all the permissible uses and thus, facilitates capitalization of the synergism among the uses appear to be more pragmatic and reasonable in our case where the optimal solution is yet to be ascertained. Thus, the problem is finding the use or mix of uses (and in what proportion) that will maximize the return from the development on the basis of the given information.

4.2.1 Empirical Modelling of the Problem

The data in the preceding subsection are used to model the HBU/HBMU problem as follows.

4.2.1.1 NLP Model

Define the following decision variables:

\[ x_{1a} = \text{Total floor area of residential development located right next to retail development} \]

The retail use is thus a negative externality.

\[ x_{1b} = \text{Total floor area of residential development located near to retail development} \]

This category is at a “safe” distance from the retail. Proximity to the retail development has favourable effect on the value of this category of residential development.

\[ x_{2a} = \text{Total floor area for retail use (excluding restaurant) in shopping mall.} \]

\[ x_{2b} = \text{Total floor area of retail development (excluding restaurant) adjoining hotel development.} \]

\[ x_{3a} = \text{Total floor area of office development located right next to retail development and/} \]
or share the same entrance. Retail development is a negative externality to the office space.

\( x_{3b} = \) Total floor area of office development located near to retail development – Office space is located at a congenial distance. Retail development becomes an amenity and thus favourably impacts the value of the office space.

\( x_4 = \) Total floor area of hotel development.

\( x_5 = \) Total floor area of restaurant development.

\( x_6 = \) Total floor area of Cineplex development.

Maximize

\[
 r_1(1 + d_{1a}(x))x_{1a} + r_1(1 + d_{1b}(x))x_{1b} + r_2(1 + d_{2a}(x))x_{2a} + r_2(1 + d_{2b}(x))x_{2b} + r_3(1 + d_{3a}(x))x_{3a} \\
+ r_3(1 + d_{3b}(x))x_{3b} + r_4(1 + d_{4}(x))x_4 + r_5(1 + d_{5}(x))x_5 + r_6(1 + d_{6}(x))x_6
\]

subject to the following constraints:

\[
 x_{1a} + x_{1b} + x_{2a} + x_{2b} + x_{3a} + x_{3b} + x_4 + x_5 + x_6 \leq 160,016 \quad (8)
\]

\[
c_1x_{1a} + c_1x_{1b} + c_2x_{2a} + c_2x_{2b} + c_2x_{3a} + c_2x_{3b} + c_4x_4 + c_5x_5 + c_6x_6 \leq 719,259,550 \quad (9)
\]

\[
 - x_{2a} - x_{2b} - x_5 \leq -5,000 \quad (10)
\]

\[
 - x_{3a} - x_{3b} \leq -64,006.4 \quad (11)
\]

\[
 - x_4 \leq -24,002.4 \quad (12)
\]

\[
 -x_6 \leq -1,000 \quad (13)
\]

\[
x_6 \leq 3,000 \quad (14)
\]

\[
0.2x_{1a} + 0.2x_{2b} - 0.8x_5 \leq 0 \quad (15)
\]

where

\[
r_1 = 12,220, r_2 = 18,840, r_3 = 10,760, r_4 = 13,750, r_5 = 13,078, r_6 = 15,850
\]

\[
c_1 = 4,500, c_2 = 4,350, c_3 = 3,950, c_4 = 5,650, c_5 = 4,350, c_6 = 3,000;
\]

\[
d_{1a}(x) = -0.15g_2(x_{2a} + x_{2b} + x_5) + 0.12g_2(x_{3a} + x_{3b}) \quad (16)
\]

\[
d_{1b}(x) = \kappa_2g_2(x_{2a} + x_{2b} + x_5) + \kappa_3g_3(x_{3a} + x_{3b}) \quad (17)
\]

\[
d_{2a}(x) = 0.25g_1(x_{1a} + x_{1b}) + 0.07g_3(x_{3a} + x_{3b}) + (0.15 \text{ to } 0.20)g_6(x_6) \quad (18)
\]

\[
d_{2b}(x) = 0.25g_1(x_{1a} + x_{1b}) + 0.07g_3(x_{3a} + x_{3b}) - (0.10 \text{ to } 0.20)g_4(x_4) + (0.15 \text{ to } 0.20)g_6(x_6)
\]
\[
\begin{align*}
    d_{3a}(x) &= 0.05 g_1(x_{1a} + x_{1b}) - 0.10 g_2(x_{2a} + x_{2b} + x_5) \\
    d_{3b}(x) &= 0.05 g_1(x_{1a} + x_{1b}) + 0.20 g_2(x_{2a} + x_{2b} + x_5) \\
    d_4(x) &= 0, \quad d_5(x) = 0, \quad d_6(x) = 0
\end{align*}
\]  
(20) (21)

where \( \kappa_2 = 0.2, \kappa_3 = 0.1, \)

\[
\begin{align*}
    g_1(t) &= g_2(t) = g_3(t) = g_4(t) = \beta \left(1 - \exp\left(-\frac{2t}{40,004}\right)\right) \\
    g_6(t) &= \beta \left(1 - \exp\left(-\frac{2t}{1,000}\right)\right)
\end{align*}
\]  
(22) (23)

\[\beta = \frac{1}{1 - \exp(-2)} = 1.1565\]

**Note:** one can also model the effect of oversupply of retail or office space by modifying the \(d's\) in the above model\(^3\).

The functions \(d_{1a}(x), \ldots, d_6(x)\) represent the fractional adjustments to the nominal returns \(r_1, \ldots, r_6\) of the properties \(x_{1a}, \ldots, x_6\) when the synergistic effects of a mixed-use development are taken into account. We model the return adjustment to a particular land use to be dependent on the floor areas of the other land uses in a concave manner as specified by the functions in (22)-(23). The function \(g_3(t)\) is chosen such that its value is equal to 1 when \(t = 40,004\), i.e., when the total office area reaches 40,004 m\(^2\), its contributions to the return adjustments on other properties reach the values given in Table 1. Similar explanations hold for the functions \(g_1(t), g_2(t)\) and \(g_4(t)\). The value of 40,004 was obtained by assuming that the return adjustments given in Table 1 are based on dividing the GFA uniformly among the 4 major types of properties: retail, residential, office, and hotel. The function \(g_6(t)\) associated with the Cineplex development is chosen such that its value is equal to 1 when \(t\) reaches the minimum stipulated area of 1000 m\(^2\).

*4.2.1.2 LP Model*

Maximize

\[
\begin{align*}
    r_1x_{1a} + r_1x_{1b} + r_2x_{2a} + r_2x_{2b} + r_3x_{3a} + r_3x_{3b} + r_4 + r_5 + r_6
\end{align*}
\]  
(24)

subject to the constraints in equations (8) – (15).

Both the LP and NLP models have the same constraints (equations 8-15). The main difference between the two models lies in the objective functions (equations 7 and 24). The objective
function for the LP model assumes either linear relationships among the related land uses or that the effect of any synergy among the land uses is encapsulated in $r_1, \ldots, r_6$ (i.e. the value per m$^2$ of the land uses. The probability of $r_1, \ldots, r_6$ reflecting the synergism of the land uses of a “white site” (as suggested by Addae-Dapaah, 2005) is very low as there was no acceptable analog for “white sites” at the time they appeared on the market. Moreover, notwithstanding the development of a few “white sites” in Singapore over the past few years, they markedly differ in location and permitted land uses to permit any meaningful comparison to extract $r_1, \ldots, r_6$ that reflect the synergism of the land uses. Thus, the effects of the interactions among the land uses $d_{1a}(x), \ldots, d_{6}(x)$ – equations 16-23 – which are a reproduction of Table 1 in NLP format, on the value of each land use are modelled in the objective function of the NLP model (equation 7). For example, $d_{1a}(x)$ signifies the impact that the synergy between $x_{1a}$ and $x_{2a}$, $x_{1a}$ and $x_{2b}$, $x_{1a}$ and $x_{3a}$, and $x_{1a}$ and $x_{3b}$, etc, has on the value of $x_{1a}$. The NLP model, being more pragmatic in capturing the synergy among the permissible uses and thus, market behaviour, is more likely to yield better and reliable results.

Equation 8 satisfies the mandatory maximum GFA while equations 11 and 12 satisfy the mandatory minimum GFA requirements imposed by the guidelines for the “white site”. Equations 10 and 12 state that a minimum amount of 5,000m$^2$ and 24,002.4m$^2$ of GFA must be allocated to retail and hotel uses respectively. Similarly, equation 9 implies that the total development cost should not exceed S$719,259,550. The non-negativity of the decision variables signifies the absurdity of constructing negative amount of residential, etc. space on the site – Each land use may be only allocated zero or positive number of floor area. It must be noted also that the modelling of the problem in terms of square metres of space, instead of buildings or rooms, negates the use of integer programming.

The above LP model has 17 unknown (n) variables (including slacks – artificial variables either to take up excess resources or make up for a shortfall in resources, i.e. GFA/funds in our case) and 8 constraints (m) to give $\binom{17}{8} = \frac{17!}{8!(17-8)!} = 24,310$ basic feasible solutions of which only one is the optimal solution. In appraisal phraseology, there are 24,310 permutations of the land uses which are legally permissible, physically possible and financially feasible of which only one is
maximally profitable and thus, constitutes the highest and best use. It is doubtful whether any valuer can know the precise number of feasible solutions (24,310) to be investigated without recourse to LP/NLP modelling. The NLP model is even more complicated than the LP model as the optimal solution of the former may or may not be an extreme point of the feasible set as discussed under sections 4.1.1 and 4.1.2.

4.3 Discussion of Results
A summary of the LP and NLP models’ results are presented in Table 2.

Table 2
The detailed MATLAB programming and results are presented in the appendix. The NLP optimal objective function (i.e. gross development value) of S$2,795,381,282⁴ is 22.04% higher than the LP objective function value of S$2,290,538,812⁴ – a difference of S$504,842,000. The difference in the optimal objective function values for the two models results from the variation in the allocation of the GFA among the permitted land uses.

According to the results of the LP model, the site should be put to a mixed-use development of retail (56,806 m² – 35.5% of GFA), restaurant (14,201 m² – 8.87% of GFA), office (64,006 m² – 40% of GFA), hotel (24,002 m² – 15% of GFA) and Cineplex (1,000 m² – 0.62% of GFA). The LP model does not make any allocation to residential use (see Table 2). The NLP model, on the other hand, allocates the GFA to residential (11,907 m² – 7.44%), retail (46,586 m² – 29.11%), restaurant (11,646 m² – 7.28%), office (64,006 m² – 40%), hotel (24,002 m² – 15%), Cineplex (1,868 m² – 1.17%) – see Table 2. The only land uses with the same allocation of GFA are hotel (24,002 m²) and office (64,006 m²) but this is mainly a function of the mandatory requirement under the guidelines. Thus, the failure of the LP model to effectively account for the synergy among the land uses results in the sub-optimal gross development value which is 22.04% lower than the value from the NLP model. In other words, the mean value of the aggregate effects of the synergies among the land uses in this particular case (i.e. the synergy premium) is S$504,842,470 (i.e. S$2,795,381,282 – S$2,290,538,812). More research is needed on this as the size of the synergy premium (even in percentage terms) could be case-specific due to the fact
that the possible permutations of permissible land uses and synergies thereof could vary from case to case.

The optimal solution for each model has $x_{1a} = 0, x_{3a} = 0$ (Table 2). This implies that one should not build residential and office developments right next to retail development(s). This is logical since residential and office developments located right next to retail development have lower returns than the same developments located slightly further away. It must be cautioned that this conclusion is solely based on economic rationality. If it is necessary, for any reason, to build residential and office units in close proximity to retail units, the requirement can be modelled as a constraint. All other things being equal, such a constraint will have a negative impact on the optimal solution. Similarly, the results for the LP and NLP models reveal that it does not make economic sense to build a retail development adjoining to hotel development given that the latter would negatively impact such a retail development. Furthermore, the budget constraint (equation 9) is active in both the NLP and LP model when the unit cost is increased by 3% or more (column 3-6 of Table 2).

When solving for the optimal solution of a LP/NLP model, one also gets the shadow price associated with each constraint. The shadow price is zero when the constraint is not active and positive when the constraint is active. The shadow price $\lambda_i$ associated with the i-th constraint indicates that if the right-hand side of that constraint is increased by a small amount $\delta$, the optimal objective function value would increase by the amount $\lambda_i \delta$. The shadow prices for various problems associated with the budget constraint (equation 9) are shown in Table 3. Thus for the problem that corresponds to the second row and third column of Table 3, an increment of $\delta$ units of dollars in the budget would increase the total return by $5.394 \times \delta$ units of dollars. This is in contrast to the average return of 3.871 ($=2784.238/719.260$) units of dollars per unit budget invested. Thus the analysis suggests that it is desirable to increase the budget where possible.

**Table 3**
The figures in Table 4 demonstrate the sensitivity of the optimal objective function value to changes in the positive effects that retail and office developments have on residential property
development in the problem. One can see from Table 4 that more residential property will be built when the positive effects of the retail and office properties on the residential property are stronger. However, the total net return increases very little (by about 2.3%) from the weakest effect ($\kappa_2 = 0.05, \kappa_3 = 0.05$) to the strongest effect ($\kappa_2 = 0.25, \kappa_3 = 0.25$).

Table 4

It must be noted that the above analysis is based on the assumption that the site is fully developed to the maximum ratio of building area to lot size as the value of any parcel of land in Singapore is, among other things, a function of the above ratio. According to Capozza and Li (1994), a development’s intensity affects construction cost and the optimal decision option. However, the impact of various design options vis-à-vis the minimum GFA has not been investigated since the minimum GFA is not a binding constraint at the optimal solution.

5. Valuation of the Site
In consonance with valuation practice in Singapore, the site may be valued via the residual method by utilizing the figures in the empirical models (equations 7-15) and the summary of the LP and NLP outputs (Table 2) as presented in Table 5

Table 5

The HBU of the site has been ascertained through the LP model to be retail (56,806 m²), office (64,006 m²), hotel (24,002 m²), restaurant (14,201 m²), and Cineplex (1,000 m²). Similarly, the HBU for the site via the NLP model is residential (11,907 m²), retail (46,586 m²), office (64,006 m²), hotel (24,002 m²), restaurant (11,646 m²), and Cineplex (1,868 m²). These optimal allocations are the bases of the objective function values for both models presented in Table 2. These objective value figures, together with car parks, form the basis of the total gross development value from which the site value is derived. Furthermore, the value and construction costs per m² of residential, retail and office, restaurant, hotel and Cineplex used for the valuation (Table 5) are in the empirical model (equations 7-15). All the significant inputs to the residual valuation model, especially the HBMU, would, at best, be an educated guess without the LP/NLP modeling of the problem.
6. Conclusion

The paper discusses the inability of the Residual method of valuation (and in general all the traditional methods of valuation including the Cash Flow method) to deal with the optimization problem involved in the valuation of a mixed-use development site to suggest that a marriage between management science and the residual methodology provides a more scientifically based, and therefore, a more supportable estimate of value. It has been argued that the synergism among the permitted uses on a mixed-use development site makes the assumption of a linear relationship somewhat simplistic. Thus, the use of the LP model to resolve the optimization problem involved in the valuation is not intuitively appealing and could lead to inaccurate valuation. To demonstrate that a NLP modelling is more intuitively appealing, and more theoretically and empirically sound than the LP model, a real mixed-use development site in Singapore was modelled via LP and NLP. The results reveal that while the NLP model allocates the site to all the six permitted uses, the LP model allocates the site to five of the uses excluding residential use. Furthermore, the NLP model (which accounts for the synergies among the permitted uses) results in a gross development value that is 22.04% higher than the LP model – a synergy premium of S$504,842,470. Similarly, the value of the site is 39.81% (S$231,600,000) higher under the NLP model than the LP model (see Tables 5a&b). This shows that the assumption of a linear relationship in the valuation of a mixed-use development site could be problematic.

However, it must be cautioned that the synergy premium, even in percentage terms, could be peculiar to the case at issue and thus, should not be considered as being of universal application. It is a function of the permissible land uses and the synergies thereof. Any site with a different configuration (permissible land uses, synergies, etc.) could result in a different synergy premium. More research is therefore required on this.

In addition to resolving the optimization problem in an efficient way (in terms of time and cost savings) to provide a scientifically testable set of significant inputs for the residual valuation model, the programming results provide sufficient relevant information for objective, elaborate and persuasive explanation of the optimal solution. Furthermore the models provide relevant information for monitoring the development of the site as well as for investment counselling.
However, the NLP model is both theoretically and intuitively more appealing, more pragmatic and yields better results than the LP model. Given its ability to resolve the optimization problem inherent in the HBMU of a mixed-use development site on which valuations are premised vis-à-vis the availability and advancement in the NLP technology, it is hoped that the valuation profession and academics, which traditionally gravitate towards finance, will embrace NLP to improve the quality of the valuation of mixed-use development sites.
Endnotes

1 At the time of writing, US$1.00 was equivalent to S$1.365

2 After subtracting the minimum floor areas for retail (5000 m²), office (64,006 m²), hotel (24,002 m²), if each of the four uses (retail including restaurant, residential, office, hotel) occupies the same amount of floor space, each use will be allocated 16,752 m² (i.e. 67,007 m² ÷ 4). Thus “S$719,259,550” is derived from 16,752 \( \sum_{i=1}^{n=4} c_i + 5,000c_2 + 64,006c_3 + 24,002c_4 \)

3 An oversupply of retail floor area may be modeled by modifying \( d_{2a}(x) \) to include a term such as \( -\delta\max(0,x_{2a} - 16.752) \) to account for the negative effect of oversupplying retail space beyond the benchmark area of 16.752×1000 m² under uniform development. Here, \( \delta \) is a suitable return adjustment to account for the negative impact of oversupply.

4 The magnitude of the gross development value of the newly completed project is very realistic in Singapore although it may appear preposterous to the western reader.
Reference


Table 1: Effects of Synergies among Decision Variables

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>$x_{1a}$</th>
<th>$x_{1b}$</th>
<th>$x_{2a}$</th>
<th>$x_{2b}$</th>
<th>$x_{3a}$</th>
<th>$x_{3b}$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
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<td>0</td>
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</tr>
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<td>$x_{2a}$</td>
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<td>0</td>
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<td>-0.10</td>
<td>0.20</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_{2b}$</td>
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<td>0.20</td>
<td>0</td>
<td>0</td>
<td>-0.10</td>
<td>0.20</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_{3a}$</td>
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<td>0.10</td>
<td>0.07</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_{3b}$</td>
<td>0.12</td>
<td>0.10</td>
<td>0.07</td>
<td>0.07</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_4$</td>
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<td>0</td>
<td>0</td>
<td>-0.10 to -0.20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_6$</td>
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<td>0</td>
<td>0.15 to 0.20</td>
<td>0.15 to 0.20</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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</table>

Note: The figures show the effect that the variables in the “rows” have on the variables in the “columns.”
Table 2: Effect of Changes in Unit Costs on Total Optimal Returns. $\kappa_2 = 0.2, \kappa_3 = 0.1$. (Objective Function Figures in 1,000,000)

<table>
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<tr>
<th>unit costs up by</th>
<th>0%</th>
<th>3%</th>
<th>6%</th>
<th>9%</th>
<th>12%</th>
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<tr>
<td>NLP Optimal Objective Value</td>
<td>2795.381</td>
<td>2784.238</td>
<td>2674.541</td>
<td>2571.085</td>
<td>2473.359</td>
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<td>Improvement over LP objective (%)</td>
<td>22.04</td>
<td>21.70</td>
<td>21.06</td>
<td>20.52</td>
<td>19.98</td>
</tr>
<tr>
<td>$x_{1b}$</td>
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<td>10.350</td>
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<td>64.006</td>
<td>64.006</td>
<td>64.006</td>
</tr>
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<td>11.692</td>
<td>10.993</td>
<td>10.342</td>
<td>9.735</td>
</tr>
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<td>3.000</td>
<td>3.000</td>
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<td>719.260</td>
<td>719.260</td>
<td>719.260</td>
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<td>55.615</td>
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<td>$x_{3b}$</td>
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Table 3: Shadow Price Associated With The Budget Constraint (top number) and Average Return (bottom number). $\kappa_2 = 0.2$, $\kappa_3 = 0.1$.

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<tr>
<th>unit costs up by</th>
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<th>6%</th>
<th>9%</th>
<th>12%</th>
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<td>unit returns up by</td>
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<td>5.394</td>
<td>5.231</td>
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<td></td>
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<td>3.871</td>
<td>3.718</td>
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<tr>
<td>15%</td>
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<td>6.016</td>
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Table 4: Changes in $\kappa_2, \kappa_3$ Versus Changes in Optimal Objective Value and $x_{1b}$.

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<th>$\kappa_2$</th>
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<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
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<td>2774.219</td>
<td>2780.756</td>
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<td>2780.629</td>
<td>2787.740</td>
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<td></td>
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<td>10.977</td>
<td>11.954</td>
<td>12.985</td>
<td>14.075</td>
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<td></td>
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<td>2795.381</td>
<td>2803.781</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>10.939</td>
<td>11.907</td>
<td>12.927</td>
<td>14.003</td>
<td>15.142</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td>2795.267</td>
<td>2803.632</td>
<td>2812.698</td>
<td>2822.504</td>
<td>2833.091</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.860</td>
<td>12.870</td>
<td>13.934</td>
<td>15.057</td>
<td>16.245</td>
</tr>
</tbody>
</table>
Table 5a: Valuation of Site Based on LP Model

<table>
<thead>
<tr>
<th>Value of New Completed Development</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function Value</td>
<td>$2,290,538,812</td>
</tr>
<tr>
<td>Car park: 628 lots @ $800 per lot</td>
<td>$502,400</td>
</tr>
<tr>
<td><strong>Total - Gross Development Value (GDV) - fval + car park</strong></td>
<td>$2,291,041,212</td>
</tr>
</tbody>
</table>

**Deduct**

| Selling expenses (3% of GDV) | $68,731,236 |
| Goods & Services Tax – GST (7% of selling expenses) | $4,811,187 |
| Goods & Services Tax (7% of GDV) | $160,372,885 |
| **Total expenses** | $233,915,308 |

**Net Development Value**

$2,057,125,904

**Deduct: Construction cost including financing cost and professional fees**

| Retail: 56,806m² @ $4,350 per m² | $247,106,100 |
| Office: 64,006m² @ $3,950 per m² | $252,823,700 |
| Restaurant: 14,201m² @ $4,350 per m² | $61,774,350 |
| Hotel: 24,002m² @ $5,650 per m² | $135,611,300 |
| Cineplex: 1,000m² @ $3,000 per m² | $3,000,000 |
| Car park: 19,500m² @ $770 per m² | $15,015,000 |
| **Sub-total** | $715,330,450 |

**Add: Developer’s profit (say 20% of GDV)**

$458,208,242

**Total**

$1,173,538,692

**Surplus for Land**

$883,587,212

**Value of Site**

| Let value of land be | $x |
| Add Stamp duty and legal fees (say 4% of land value) | 0.04$x |
| .: Land value plus stamp duty and legal fees | 1.04$x |
| Add GST (7% of 1.04$x) | 0.0728$x |
| Sub-total | 1.1128$x |

**Add**

| Cost of finance (say 6% per annum for 5 years) c | 0.3763774$x |
| Property tax d | 0.0295195$x |
| **Total Land Cost** | 1.5186969$x |

Thus, 1.5186969$x = $883,587,212 (i.e. Surplus for land)

.: $x$ (i.e. value of site) = $883,587,212/1.5186969

$x$ = $581,806,165. Thus, the site is valued at say $581,800,000

---

a Car parks were not included in the optimization modeling of the problem because they are not included in the minimum and maximum GFA requirements of the planning guidelines for the “white” site.

b Stamp duty and legal expenses related to the purchase of a parcel of land in Singapore account for 2% - 5% of the value of the land.
Cost of finance is based on the assumption that the development will take 5 years to complete and the cost of debt is 6% per annum (annual compounding).

Any land under development in Singapore is subject to property tax. The property tax is a function of “annual value” which, under proviso (f) to s2 of the Property Tax Act (Cap 254, 1985 edition), is 5% of the value of vacant land. The current property tax rate is 10%. Thus, if the value of the site is X, the “annual value” is 0.05X, and the property tax payable for each year is 0.005X (i.e. 0.05X x 10%). This is payable semi-annually in advance (i.e. two equal installments of 0.0025X per annum. Therefore, the total amount of property tax payable over the 5-year holding period and interest accumulation thereof is calculated as:

\[ 0.0025 \times \left( \left( \left( 1 + \frac{i}{2} \right)^{2 \times 5} - \left( 1 + \frac{i}{2} \right)^{5} \right) \right) \]  

Although the proceeds of sale are receivable on completion of the development at a future date, it would be incorrect to discount the proceeds to their present-day value since “the cost of holding the property is taken as a cost of the development. Therefore to discount the proceeds of sale would be double counting.” (Johnson et al. 2000:166)
Table 5b: Valuation of Site Based on NLP Model

<table>
<thead>
<tr>
<th>Value of New Completed Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function Value (Exhibit 2)</td>
</tr>
<tr>
<td>Car park^a: 628 lots @ S$800 per lot</td>
</tr>
<tr>
<td><strong>Total - Gross Development Value (GDV)</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deduct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling expenses (3% of GDV)</td>
</tr>
<tr>
<td>Goods &amp; Services Tax – GST (7% of selling expenses)</td>
</tr>
<tr>
<td>Goods &amp; Services Tax (7% of GDV)</td>
</tr>
<tr>
<td><strong>Total expenses</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Net Development Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>S$2,510,423,958</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deduct: Construction cost including financing cost and professional fees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail: 46,586m^2 @ S$4,350 per m^2</td>
</tr>
<tr>
<td>Office: 64,006m^2 @ S$3,950 per m^2</td>
</tr>
<tr>
<td>Residential: 11,907m^2 @ S$4,500 per m^2</td>
</tr>
<tr>
<td>Restaurant: 11,646m^2 @ S$4,350 per m^2</td>
</tr>
<tr>
<td>Hotel: 24,002m^2 @ S$5,650 per m^2</td>
</tr>
<tr>
<td>Cineplex: 1,868m^2 @ S$3,000 per m^2</td>
</tr>
<tr>
<td>Car park: 19,500m^2 @ S$770 per m^2</td>
</tr>
<tr>
<td><strong>Sub-total</strong></td>
</tr>
<tr>
<td><strong>Add</strong>: Developer’s profit (say 20% of GDV)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surplus for Land</th>
</tr>
</thead>
<tbody>
<tr>
<td>S$1,235,302,522</td>
</tr>
</tbody>
</table>

Value of Site

<table>
<thead>
<tr>
<th>Let value of land be</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Add</strong>: Stamp duty and legal fees^b (say 4% of land value)</td>
<td>0.04 x</td>
</tr>
<tr>
<td><strong>∴</strong> Land value plus stamp duty and legal fees</td>
<td>1.04 x</td>
</tr>
<tr>
<td><strong>Add</strong>: GST (7% of 1.04 x)</td>
<td>0.0728 x</td>
</tr>
<tr>
<td><strong>Sub-total</strong></td>
<td>1.1128 x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Add</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of finance (say 6% per annum for 5 years)^c</td>
</tr>
<tr>
<td>Property tax^d</td>
</tr>
<tr>
<td><strong>Total Land Cost</strong></td>
</tr>
</tbody>
</table>

Thus, 1.5186969 x = S$1,235,302,522 (i.e. Surplus for land)

∴ x (i.e. value of site) = S$1,235,302,522 / 1.5186969

x = S$813,396,355. Thus, the site is valued at say **S$813,400,000**

**Source:** Author

- Value of site under NLP model is 39.81% higher than that under LP model
Appendix

global unit_return D

GFA = 160.016;
c = [4.500; 4.500; 4.350; 4.350; 3.950; 3.950; 5.650; 4.350; 3.000];
A = [1, 1, 1, 1, 1, 1, 1, 1, 1;
c(1), c(2), c(3), c(4), c(5), c(6), c(7), c(8), c(9);
0, 0, -1, -1, 0, 0, 0, -1, 0;
0, 0, 0, 0, -1, 0, 0, 0, 0;
0, 0, 0, 0, 0, -1, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, -1;
0, 0, 0, 0, 0, 0, 0, 0, 1;
0, 0, 0.2, 0.2, 0, 0, -0.8, 0]
b = [GFA; 719.25955; -5.0; -64.0064; -24.0024; -1.000; 3.000; 0.000]
numvar = length(c);
LB = 0*ones(numvar,1);
UB = inf*ones(numvar,1);
15.850];
kap2 = 0.2; kap3 = 0.1;
D = [0, 0, -0.15, -0.15, 0.12, 0.12, 0, 0, 0;
0, 0, kap2, kap2, kap3, kap3, 0, 0, 0;
0.25, 0.25, 0, 0, 0.07, 0.07, 0, 0, 0.175;
0.25, 0.25, 0, 0, 0.07, 0.07, -0.15, 0, 0.175;
0.05, 0.05, -0.1, -0.1, 0, 0, 0, 0, 0;
0.05, 0.05, 0.2, 0.2, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0;
0, 0, 0, 0, 0, 0, 0, 0, 0];

%%
%% LP model
%%

options = optimset('Display','off');
[LPsol, LPObj, flag, output, lambda] = linprog(-unit_return, A, b, [], [], LB, UB, [], options);
LPsol(3) = LPsol(3)+LPsol(4); LPsol(4)=0;
LPsol(6) = LPsol(6) + LPsol(5); LPsol(5) = 0;
LP.objective = -LPobj;
LP.solution = LPsol;
LP.dual = lambda.ineqlin;
LP.slack = A*LPsol - b;

%%
%% NLP model
%%
x0 = LPsol;
options = optimset(options,'GradObj','off');
[NLPsol,NLPobj,flag,output,lambda] =
fmincon('NLPmodelfun_exp',x0,A,b,[],[],LB,UB,[],options);
NLP.objective = -NLPobj;
NLP.solution = abs(NLPsol);
NLP.dual = lambda.ineqlin;
NLP.slack = A*NLPsol - b;