Outsourcing and Computers: Impact on Urban Skill Level and Rent

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Abstract

Cities in the U.S. with a higher initial share of college graduates have had a greater subsequent increase in this share over the past two decades. Concurrently, housing prices have grown faster in these skilled cities. This paper argues that the diffusion of computers and outsourcing may partly explain these two phenomena. In the presented model, skilled workers are more productive in skilled cities and need unskilled support services. The cities’ unskilled workers can perform the support services, but when it is cheaper, such services can be undertaken by computers or outsourced to less-skilled cities. New technologies facilitating computerization and outsourcing can increase the skill share and housing prices in skilled cities relative to less-skilled cities, under reasonable assumptions. The basic economics is that the new technologies diminish the demand for unskilled workers in skilled cities and permit skilled workers to earn higher wages, which in turn increases the supply of skilled workers in skilled cities and drives up housing prices. Empirically, this paper documents five stylized facts that the theory can rationalize. Particularly important is rising skill premium in skilled cities relative to less-skilled cities, which supports a production theory involving shifts in labor demand.

JEL Classification: J23, R12, R2 and R3

Keywords: Computerization; domestic outsourcing; migration; technological change; technology-skill complementarity
1 Introduction

There are two pronounced trends for U.S. cities. First, cities like Boston and New York with a higher initial skill share, which is defined as the share of workers having a bachelor’s degree, have had a greater subsequent increase in this share (Glaeser, 1994; and Berry and Glaeser, 2005). Second, these skilled cities, which have a higher initial skill share, have also undergone faster growth in housing prices (Glaeser, 2000). While various explanations, such as a consumer theory featuring an increased supply of skilled workers in the skilled cities, can connect these two trends, this paper proposes a production theory involving technological changes and shifts in labor demand.

The proposed theory is motivated by two salient technological changes: a decrease in communication costs (Doms, 2005) facilitating outsourcing⁠¹ and a decrease in computing prices (Gordon, 1990; and Jorgenson, 2001) triggering computerization, i.e., office automation. The theory suggests that both technological changes can increase the skill share and housing prices in skilled cities relative to less-skilled cities under reasonable assumptions. The theory is also consistent with three additional stylized facts. First, unskilled business support jobs are increasingly concentrated in less-skilled cities, and this is not yet mentioned in literature. Second, computers are more intensively used in skilled cities. These two facts make outsourcing and computerization potential explanations for why skilled and less-skilled cities are increasingly dissimilar. Third, skill premium increases faster in skilled cities, which is also important as this supports a theory involving labor demand shifts.

The model comprises two cities and has two essential ingredients. First, the cities differ in their productivity. Second, skilled and unskilled workers are complements, while technology and unskilled workers are substitutes: To produce output, skilled workers need various kinds of support services, which can be performed by unskilled workers or computers or acquired through outsourcing. The outcome of the above assumptions is the following. Without the options of outsourcing and computerization, to put skilled workers in the more productive city

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¹In this paper, outsourcing is defined as the location separation of production tasks that used to be performed in the same city. Outsourcing occurs whenever a plant hands over some of its tasks to a plant in another city, even when the two plants are owned by the same firm. This is vertical disintegration at the plant level.
requires the presence of unskilled workers in the same city. But housing is expensive in this city, and these unskilled workers demand higher wages to compensate for expensive housing. Thus, firms in the more productive city have less incentive to use local unskilled workers to perform the support services when outsourcing or computerization is possible, compared to other firms in the less productive city. This is why the more productive city is also the skilled city.

New technologies that lower prices of computers and communications can increase the skill share of the skilled city relative to the less-skilled city, as the new technologies decrease both the demand and supply of unskilled workers relative to skilled workers in the skilled city. The relative demand decreases due to technology-skill complementarity. The relative supply decreases because a greater number of skilled workers choose living in the skilled city as the technological changes increase skilled workers’ marginal productivity more in the skilled city through lowering prices of the support services. Also critical is skilled city’s higher unskilled wages that are necessary for the new technologies to trigger more computerization in the skilled city relative to the less-skilled city and to lead to outsourcing to the less-skilled city.

The reason why the new technologies can also increase housing prices in the skilled city relative to the less-skilled city is more subtle. In fact, one might expect that the new technologies would do the opposite, as they expect skilled city’s unskilled wages would decrease and unskilled workers would move out. However, the model shows that the relative housing prices can go up under certain conditions. Particularly, the degree of labor mobility plays an important role.

The impact of the new technologies will be bigger if it is assumed that the city’s skill share can increase the city’s total factor productivity. Empirical evidence underlying this assumption is strong. Rauch (1993) shows that each additional year of city’s average level of education can raise productivity by 2.8%, and Moretti (2004) shows that a one-percentage-point increase in the skill share can increase productivity by about 6%.

Why is it interesting to document and explain the mentioned urban patterns? The literature has found that skilled cities perform far better economically than less-skilled cities, primarily because skilled cities experience faster increase in productivity, which can partly translate into
faster housing price growth (Glaeser and Saiz, 2004). Since the patterns were pronounced in the past few decades, it is nature to wonder the role that outsourcing and computerization could play in making these changes. Although the literature of outsourcing in international trade and computerization in labor economics are both matured, integrating the notions of outsourcing and computerization into the literature on skilled cities has received relatively less attention. Therefore, it is rewarding to formulate a theory that relates computerization and domestic outsourcing to skilled cities and document supporting facts.

This paper extends the literature on domestic-outsourcing by venturing into analysis of local labor and housing markets. Currently, the literature focuses on locations and structures of firms. Duranton and Puga (2005) show that decreased communication costs have enabled firms to separate different tasks at different locations. This changes the clustering of firms from the previous pattern of sectoral specialization to the current pattern of functional specialization—management function is concentrated in commercial cities, and production function is concentrated in manufacturing cities. Rossi-Hansberg et al. (2009) are also concerned with geographic separation of firms’ production tasks. In their model, lower communication costs result in concentration of management in the city center and concentration of production in the suburbs. Their work adds to the literature by rationalizing functional specialization within a city and providing empirical evidences. As for my paper, the model also predicts functional specialization—skilled jobs are more concentrated in one city and support jobs in another. Nevertheless, this paper makes a contribution by pushing the literature on domestic outsourcing further through examining how domestic outsourcing may interact with local labor markets and housing markets.

This paper also adds to the literature on computerization. There is a large volume of research that examines relationships between computers and skills in the national economy, but it would be interesting to extend the literature into an urban or regional context and examine whether relationships between computers and skills differ across localities. The research of Beaudry et al. (2020) finds that, at the metropolitan level, the initial skill share has a positive impact on wage and housing price growth, but, at the city and municipal levels, the relationship between the initial skill share and housing price change is driven by low-skilled cities.
al. (2006) is in this direction and is related to mine, as they examine how new PC technologies increase local demand for skilled workers and how local supply of skilled workers affects the adoption of new PC technologies. My research is different, because I assume mobile labor and am able to study how the new technologies induce migration and affect urban skill level and rent.

This paper does not attempt to argue that outsourcing and computerization are the only or a most important hypothesis that explains faster increase in the skill share and housing prices of skilled cities. Alternative explanations may also account for the changes. For instance, Berry and Glaeser (2005) suggest that skill-biased innovations increase skilled firms’ demand for skilled workers. Since these firms are concentrated in skilled cities, these cities may become increasingly and disproportionately skilled. Nevertheless, given the dramatic decrease in prices of computers and communications, it is interesting to examine whether outsourcing and computerization can have an impact.

The remainder of this paper is organized as follows. Section 2 illustrates the stylized facts. Section 3 presents a simple model that uses essential elements to highlight effects of outsourcing on the skill share and housing prices. However, this simple model cannot incorporate computerization and cannot predict the faster increase in the skill premium in skilled cities. Therefore, Section 4 introduces a richer model. Section 5 discusses the equilibrium, and Section 6 analyzes effects of outsourcing and computerization. Section 7 presents a simulation exercise that illustrates how the effects depend upon parameter values. Section 8 concludes.

2 Stylized Facts

This section presents five stylized facts on cities for the period of 1980 to 2000, using the cities’ skill share in 1980 as the initial condition. The data sources are Decennial Censuses. Workers are private employed wage and salary earners, and cities are metropolitan areas with at least a quarter million people in 1990. These metropolitan areas generally follow the 1990 MSA/CMSA/NECMA definition.
Fact 1: Housing prices increase faster in skilled cities. Figure 1 plots the 1980 skill share for each city against the logarithmic change of real housing prices over the next two decades. The correlation is 40 percent. Housing prices grew faster in skilled cities compared to less-skilled cities. On average, a one-percentage-point increase in the initial skill share increased the growth of real housing prices by 1.7 percent.

![Figure 1: Housing prices increase faster in skilled cities.](image)

This fact is not new. Glaeser and Saiz (2004) show that a one percentage point increase in the initial skill share can increase housing prices by 0.9 to 2.3 percent within a decade, controlling for housing prices, climate, industrial composition, economic condition, and year-and location-specific fixed effects. In Shapiro (2006), a significant positive impact of initial skill share on the growth of housing rental prices is also found. The impact remains robust, when the presence of land-grant colleges is used as an instrument for the cities’ initial skill share in the estimation that addresses possible reverse causality and omitted variable bias.  

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3The average annual rent is calculated for each metropolitan area. The calculation includes renter-occupied housing units with cash rent and owner-occupied housing units that are non-farm, non-commercial, single family houses on less than 10 acres of land. House values are converted to annual rent using the 7.85% discount rate common in the literature, e.g., Gyourko and Tracy (1991).

4The presence of land-grant colleges is an appealing identifying variation for the skill share (concentration of college graduates) in IV estimation examining the causal effect of the skill share on urban growth. Moretti (2004) and Shapiro (2006) provide evidences and argue that the presence of land-grant colleges satisfies the exclusion restriction of IV and does not correlate with unobservables such as geographic location, urban demography and local market conditions, as the presence is predetermined by Federal Acts in the 19th century. They also show that the presence of land-grant schools increases the rate of college attainment, but not vice versa. Thus, the IV estimation is not subject to reverse causality and omitted variable bias.
Fact 2: Skill share increases faster in skilled cities. Figure 2 plots the 1980 skill share against the increase of this share over the next two decades. The correlation is 55 percent. Skilled cities were more successful in attracting skilled workers. On average, an extra one percentage point in the 1980 skill share is associated with a 0.5 percentage point increase in the skill share over the next two decades.

![Graph showing the correlation between 1980 skill share and the change in skill share over the next two decades. The regression line is given by y = 0.02 + 0.49x, with an R-squared of 0.30.]

Berry and Glaeser (2005) examine this relationship and confirm a significant positive impact of the initial skill share on the growth of the skill share. The estimated impact is even stronger, when they use the historical number of colleges per capita as an instrument for the initial skill share. The authors also find that, before 1980, cities grew by attracting both skilled and unskilled workers, but, after 1980, cities grew by attracting skilled workers.

Looking at Figures 1 and 2, one might think that the correlations are primarily driven by a few most highly skilled metropolitan areas, but an examination which excludes these areas finds that the correlations are still considerable\(^5\). Nevertheless, one should not treat these metropolitan areas as excludable outliers, because they account for a substantial proportion of U.S. population\(^6\).

Fact 3: Unskilled business support jobs are increasingly concentrated in less-skilled cities

Fact 3 is new and not yet mentioned in literature. Here, I examine the changes in the

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\(^5\)The examination excludes the five most highly skilled metropolitan areas: Washington CMSA, Boston CMSA, San Francisco CMSA, Denver CMSA, and New York CMSA. The resulted correlations are 30% and 46% for Facts 1 and 2, respectively.

\(^6\)The five most highly skilled metropolitan areas weighted 16% of U.S. total population in 2000.
geographic concentration of office and administrative support jobs. The SOC codes classify occupations into seven major categories. One of these categories is office and administrative support, which accounted for roughly 17 percent of U.S. total employment between 1980 and 2000. These jobs are unskilled because most of them do not require a bachelor’s degree. In 2000, only 13 percent of office and administrative support workers had a bachelor’s degree, while 23 percent of American workers had the degree.

The geographic concentration of these support jobs has been shifted from skilled cities to less-skilled cities. To show this, I calculate the location quotients of the support jobs in 1980 and 2000 for each city. The location quotient is a ratio measuring the geographic concentration of activities. A city’s location quotient of the support jobs is the city’s share of U.S. support workers relative to the city’s share of U.S. total employment. If the support jobs are evenly distributed across cities, then every city will have a ratio that equals one. A city is more concentrated with the support jobs if the ratio is greater than one. The higher is the ratio, the higher is the concentration of the support jobs.

Figure 3 plots the 1980 skill share against the logarithmic change of the location quotient of the support jobs over the next two decades. Strikingly, the correlation was -68 percent. Highly skilled cities, including Boston, New York City, San Francisco, and Washington D.C., all had a 20 percent decrease in the location quotient, while less-skilled cities like El Paso had a 20 percent increase in the location quotient.

Figure 3: Unskilled business support jobs are increasingly concentrated in less-skilled cities

Table 1 includes a set of OLS regressions on the relationship between the initial skill share
and the subsequent decennial growth in the location quotient of the support jobs. To present this table, I include data for the year 1990. Columns 1 and 2 show the raw impact of the initial skill share on the later growth in the location quotient for the 1980s and 1990s, respectively. In Columns 3 and 4, the added control is the logarithm of city size, which may be correlated with the initial skill share and subsequent growth. Nevertheless, the impact of the city size is insignificant. Controls for industrial employment growth are added in Columns 5 and 6, since the growth of the location quotient may be due to the growth of a particular industry that uses the support workers more intensively or less intensively. The regression in Column 7 pools data of the two decades to include city fixed effects, which may reflect unobservable characteristics such as city specific policies. All of the regressions show a significant negative impact of the initial skill share on the later growth in the location quotient of the office and administrative support jobs. This negative impact is particularly more pronounced in the 1990s.

### Table 1: Initial skill level and change in support job LQ

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill share at t-10</td>
<td>-0.72*** (0.12)</td>
<td>-0.86*** (0.09)</td>
<td>-0.62*** (0.15)</td>
<td>-0.86*** (0.11)</td>
<td>-0.59*** (0.16)</td>
<td>-0.81*** (0.09)</td>
<td>-0.80** (0.35)</td>
</tr>
<tr>
<td>Log population at t-10</td>
<td>-0.01 (0.01)</td>
<td>0.00 (0.01)</td>
<td>-0.01 (0.01)</td>
<td>-0.01 (0.01)</td>
<td>-0.01 (0.01)</td>
<td>-0.02 (0.01)</td>
<td></td>
</tr>
<tr>
<td>Industrial growth</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>City fixed effect</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>132</td>
<td>132</td>
<td>132</td>
<td>132</td>
<td>132</td>
<td>132</td>
<td>264</td>
</tr>
<tr>
<td>R square</td>
<td>0.20</td>
<td>0.42</td>
<td>0.21</td>
<td>0.42</td>
<td>0.41</td>
<td>0.62</td>
<td>0.70</td>
</tr>
</tbody>
</table>

**Note:**
Standard error in parentheses

*** Significant at 1 percent level  ** Significant at 5 percent level

**Fact 4: Skilled cities use computers more intensively**  Doms and Lewis (2006) and Lewis (2006) examine Fact 4 using a firm-level dataset in which the exact number of computers owned by each firm is observable. Figure 4, which is from Lewis (2006), plots the share of workers with some college education in 1980 against the number of computers per worker in 2000. Since it controls industrial composition and firm size, Fact 4 is not simply because skilled
workers use computers. Even in a skilled industry, computers are used more intensively in skilled cities compared to less-skilled cities.

Figure 4: Skilled cities use computers more intensively.

The research of Doms and Lewis (2006) views that computers are used more intensively in skilled cities because there is an agglomeration benefit that helps with inventing computer-intensive technologies. My interpretation is more in line with the literature of technology-skill complementarity, where computers substitute for unskilled workers and complement skilled workers. Firms in skilled cities use computers, as well as outsourcing, more intensively because unskilled workers are more expensive there.

Fact 5: Skill premium increases faster in skilled cities  Figure 5 plots the 1980 skill share against the logarithmic change of skill premium over the next two decades. In this paper, city’s skill premium is defined as the ratio of the city’s average skilled wage relative to the average unskilled wage. On average, an extra one percentage point in the initial skill share increased the growth of the skill premium by 0.45 percent over the next two decades. The correlation is only 22 percent because there are outliers in the top left corner. However, all these outliers are small cities with fewer than a half million people. Taking population weight into account, the correlation is 38 percent.
Empirical literature suggests that workers earn a wage premium in skilled cities. Recently, researchers start considering whether this premium has increased overtime and whether it has increased more for skilled workers. Berry and Glaeser (2005) show that this premium has increased for skilled workers but not for unskilled workers. This finding implies that the skill premium has increased in skilled cities relative to less-skilled cities. This implication is confirmed in Beaudry et al. (2006). Controlling various city characteristics, they find that the ratio of college- to high-school-educated workers in 1980 has a significant positive impact on the growth of the return to skill over the next two decades.

**Correlations between computer prices and the stylized facts**  This paper proposes a production theory that predicts the above stylized facts. Specifically, the paper argues that it is the decline in prices of computers and communications that shape these facts. Thus, it would be helpful to examine correlations between the prices and the facts. Such an examination is possible for computer prices and Facts 1, 2, 3, and 5, using BEA’s quality adjusted computer price index\(^7\), IPUMS-CPS, and OFHEO’s Metropolitan Housing Price Indexes\(^8\). For prices

\[^7\]Gordon (1990) argues that a computer price index must be adjusted to computer quality, such as speed and memory, which has been consistently and rapidly improved. In producing a quality adjusted index, a hedonic pricing model with year dummies is useful, and the estimation often disaggregates the sample into adjacent periods and applies a regression to each period, so that the implicit price of each quality characteristic can also vary. BEA and BLS have adopted this method to produce quality adjusted price indexes for computers and peripherals.

\[^8\]The average housing price of each metropolitan area for each individual year is calculated by using the Housing Price Indexes and the home value data of Census 2000.
of communications, the lack of a quality adjusted price index over a long span hinders this correlation check.

To examine correlations between computer prices and the stylized facts, a two stage procedure is performed for each of the four variables—housing prices, skill share, location quotient of business support jobs, and skill premium. Firstly, the variable is regressed against the skill share in 1986, for each individual year from 1986 to 2004\(^9\). This gives 19 coefficient estimates of the initial skill share. Then, the values of these 19 estimates are regressed against computer prices.

Figure 6 presents the result of each of the four second-stage regressions. The results lend support to the theory. Panel A is on housing prices. It shows that housing prices would become more expensive in skilled cities relative to less-skilled cities when computer prices decrease. In 1987, the logarithmic prices of computers were 2.9, and, according to the estimates from the first stage regression, housing prices would be only 3.6 percent higher if the cities’ 1986 skill share was one percentage point higher. In 2004, the logarithmic computer prices were 0.16, and housing prices would be 4.7 percent higher if the 1986 skill share was one percentage point higher. On average, a one percent decrease in computer prices would significantly lead to a 0.36 unit increase in the coefficient estimate of the 1986 skill share obtained from the first stage regression. This result suggests that the continuous decrease in computer prices could cause housing prices to grow faster in skilled cities during the period.

\(^9\)The examination is for the period of 1986 to 2004, as this is the period for which a large number of metropolitan areas can be consistently identified in CPS data and can be matched with OFHEO’s Housing Price Indexes for metropolitan areas using MSA/PMSA definition. The examination only considers a total number of 49 metropolitan areas that had more than 200 private employed wage and salary earners sampled in the March CPS Survey in 1986, because the Census Bureau warns that estimates for smaller metropolitan areas are subject to relatively large sampling variability. I did perform the estimation which includes every identifiable metropolitan area. All results bear the same patterns as those in the main text, except the one on the skill share. The reason for this discrepancy is quite obvious. A city that gets a high skill share in the initial year due to sampling variability is less likely to get an even higher skill share in the later years.
The results of the other three second-stage regressions also support the theory. Panel B shows that a one percent decrease in computer prices would significantly lead to 0.04 units of increase in the coefficient estimate of the 1986 skill share. This suggests that skilled cities would become even more skilled relative to less-skilled cities when computer prices decrease, and a continuous decrease in prices could make the skill share to increase faster in skilled cities. Panels C and D show that a one percent decrease in computer prices would change the coefficient estimates of the 1986 skill share for the location quotient of the support jobs and the skill premium by -0.48 and 0.83 units, respectively. This suggests that skilled cities would experience deconcentration of the support jobs and obtain a higher skill premium relative to less-skilled cities when computer prices decrease, and a continuous decrease in prices could make the support jobs to be increasingly concentrated in less-skilled cities and the skill premium to increase faster in skilled cities.
3 Simple Model

This section first presents a simple model, whose setup also serves as the framework of a richer model introduced later. Then, it compares the equilibria of two cases: (i) outsourcing is impossible, and (ii) outsourcing is frictionless. The comparison shows that a technological change that facilitates domestic outsourcing is able to increase the skill share and housing prices in skilled cities relative to less-skilled cities. The comparison focuses on key conditions, but the appendix includes more details on the equilibrium of each case.

3.1 Model Setup

The model is a small open economy with two representative cities. They are the primary city and the secondary city which are denoted by the subscripts 1 and 2, respectively. These two cities have the same housing supply function:

\[ Q_j^S = r_j^\chi \]  

(1)

where \( Q_j^S \) is the quantity of housing supplied in city \( j \), \( r_j \) is the rent in city \( j \), and \( \chi \) is the elasticity of supply.

The model economy has one unit of workers. \( \phi_h \) units of these workers are skilled, and the remaining \( \phi_l \) units are unskilled. The subscripts \( h \) and \( l \) denote the skilled and the unskilled, respectively. The workers first decide between living in the primary city and the secondary city. Then, they participate in their local labor market for wages denoted by \( w_{t,j} \), where \( t \in \{ h, l \} \).

The workers’ utility function is assumed to be\(^{10}\)

\[ u_{t,j} = q_{t,j}^\alpha c_{t,j}^{1-\alpha} \]

\(^{10}\)The assumption of Cobb-Douglas preferences is empirically justified by Morris and Ortalo-Magne (2008), who use Decennial Censuses to show that the expenditure share on housing is remarkably constant across metropolitan areas and over time.
and their budget constraint is

\[ r_j q_{t,j} + c_{t,j} = w_{t,j} \]

where \( \alpha \in (0, 1) \), and \( q_{t,j} \) and \( c_{t,j} \) are the consumption on housing and tradable goods, respectively. While \( r_j \) is determined endogenously, the price of the tradable goods is set to one.

The production function of the tradable goods is

\[ y_j = A_j h_j^\psi l_j^{1-\psi} \]

where \( h_j \) and \( l_j \), respectively, are the numbers of skilled and unskilled workers employed by the representative producer in city \( j \), and \( \psi \in (0, 1) \). The producer has an option to outsource unskilled work to the other city, but, due to communication friction, outsourcing incurs a cost, which is either infinite or zero in later analysis. The production technology is abstract from the option of outsourcing the skilled work. This is without lose of generality since the technology is constant return to scale—outsourcing a skilled worker is equivalent to the relocation of a firm that employs a single skilled worker and outsources the unskilled work.

In the above production function, \( A_j \) is the total factor productivity of city \( j \). It is assumed that

\[ A_j = A_{j*} e^{\gamma \frac{H_j}{H_j+L_j}} \]  \hspace{1cm} (2)

where \( \gamma \) is the parameter of human capital spillovers, and \( \frac{H_j}{H_j+L_j} \) is city \( j \)’s skill share, as \( H_j \) and \( L_j \) denote the city’s skilled and unskilled population, respectively. This function allows a one percentage point increase in the skill share to increase \( A_j \) by \( \gamma \) percent and is in line with the literature such as Moretti (2004). Additionally, \( A_{j*} \) is a positive number assumed exogenously, and \( A_1 > A_2 \).

Lastly, there are several market clearing conditions for the housing and labor markets. While
local labor market clearing conditions will be stated later, the rest of the conditions are

\[ Q_j^S = q_{h,j}H_j + q_{l,j}L_j \quad j = 1, 2 \]  \hspace{1cm} (3)

\[ \phi_h = H_1 + H_2 \]

\[ \phi_l = L_1 + L_2 \]

3.2 Analysis

Consider first a situation where outsourcing is impossible as it will incur infinite friction. In this case, the production needs the presence of skilled and unskilled workers in the same place. Thus, both cities are inhabited by both types of workers, and this requires each type of workers to be indifferent between the two cities in equilibrium. As the result,

\[ \frac{w_{h,1}}{w_{h,2}} = \left( \frac{r_1}{r_2} \right)^\alpha = \frac{w_{l,1}}{w_{l,2}} \] \hspace{1cm} (4)

must hold, because the equalization of each type of the workers’ indirect utility for the two cities results in the equalization of the primary city’s wage premiums of skilled and unskilled workers.

The first order conditions of the two producers’ profit maximization problems imply a one-to-one relationship between the skill premium and the skill mix:

\[ \frac{w_{h,j}}{w_{l,j}} = \frac{\psi l_j}{1 - \psi h_j} \] \hspace{1cm} (5)

As outsourcing is impossible, \( l_j = L_j \) and \( h_j = H_j \) must hold to clear the local labor markets. Since (4) indicates that both cities have the same skill premium, (5) then implies that both cities have the same skill mix. As the result, the skill share \( \frac{H_j}{L_j + H_j} \) is equal to \( \phi_h \) in both cities.

The first order conditions of the profit maximization also imply

\[ w_{h,j} = \psi (1 - \psi) \psi \psi^{1 - \psi} A_j \psi^{1 - \psi} \psi^{1 - \psi} \] \hspace{1cm} (6)
This equation, together with (2) and (4), implies that the productivity of the primary city relative to the secondary city determines the primary city’s wage premium, and the condition:

$$\frac{A_1}{A_2} = \frac{w_{h,1}}{w_{h,2}} = \frac{w_{l,1}}{w_{l,2}} = \left(\frac{r_1}{r_2}\right)^{\alpha}$$  \(7\)

must hold in equilibrium.

Consider a second situation where outsourcing is frictionless. If \(e^r < \frac{A_1}{A_2} - \left(\frac{\psi}{1-\psi}\right)^{\frac{\alpha\psi}{\lambda+\mu}} e^{-\gamma}\), then, in equilibrium, outsourcing makes unskilled workers all live in the secondary city, but skilled workers are in both cities. This is justified in Proposition 1 below, and the effect of releasing the parametric assumption is discussed after the proof. Here, note that the condition:

$$\left(\frac{A_1}{A_2}\right)^{\frac{1}{\psi}} = \frac{w_{h,1}}{w_{h,2}} = \left(\frac{r_1}{r_2}\right)^{\alpha}$$  \(8\)

must hold in equilibrium. The first equality is from (6) and the fact that, with frictionless outsourcing, the producers in the two cities both pay \(w_{l,2}\) to access to secondary city’s unskilled workers. The second equality is from equalizing the skilled workers’ indirect utility for the two cities. The primary city’s wage premium of unskilled workers does not appear in the above condition, because this premium is not defined, given unskilled workers all live in the secondary city in equilibrium.

Comparing (7) and (8), one can find that, from the first case with impossibility of outsourcing to the second case with frictionless outsourcing, the rent in the primary city increases relative to the secondary city. This is because outsourcing makes the primary city more skilled and more productive relative to the secondary city: When outsourcing empties the unskilled jobs and workers out of the primary city, the skill share is 1 in the primary city and \(\frac{H_2}{H_2+\phi_l}\) in the secondary city. Thus, \(\left(\frac{A_1}{A_2}\right)^{\frac{1}{\psi}} > \frac{A_1}{A_2}\), given (2).

**Proposition 1** If outsourcing is frictionless and \(e^r < \frac{A_1}{A_2} - \left(\frac{\psi}{1-\psi}\right)^{\frac{\alpha\psi}{\lambda+\mu}} e^{-\gamma}\), then, in equilibrium, unskilled workers are all in the secondary city due to outsourcing but skilled workers are in both cities.
Proof. First, outsourcing must occur in equilibrium. Suppose not. Then, (7) must hold. As \( \frac{w_{t,1}}{w_{t,2}} = \frac{1}{\bar{r}_2} > 1 \), the primary city’s producer can strictly increase profit by outsourcing an \( \varepsilon \) amount of unskilled work to the secondary city, which is a contradiction.

Second, incomplete outsourcing, the situation that a producer outsources some but not all of its unskilled work, cannot constitute an equilibrium. Suppose not. Then, \( w_{t,1} = w_{t,2} \) must hold to keep the producer indifferent between performing unskilled work in-house and outsourcing the work. Because \( r_1 = r_2 \) must also hold to keep unskilled workers indifferent between the two cities and \( A_1 > A_2 \) always hold given the parametric assumption, it is immediately seen that skilled workers strictly prefer the primary city and all choose to live there. Moreover, skilled workers spend \( \alpha \psi \) proportion and unskilled workers spend another \( \alpha (1 - \psi) \) proportion of the economy’s output on housing, but the two cities have the same housing prices. This implies \( \psi < \frac{1}{2} \), so that the primary city can accommodate some unskilled workers in addition to \( \phi_h \) units of skilled workers. However, the parametric assumption requires \( \psi > \frac{1}{2} \), which is a contradiction.

Third, consider complete outsourcing, where all unskilled work of one city is outsourced to the other. Suppose skilled workers are all in the secondary city. Then, complete outsourcing will put all unskilled work in the primary city, and unskilled workers must all prefer living there. If \( \frac{\bar{r}_1}{\bar{r}_2} > 1 \), \( \varepsilon \) unskilled workers can move to the secondary city and be strictly better off as they can earn the same wage but pay less rent. If \( \frac{\bar{r}_1}{\bar{r}_2} \leq 1 \), \( \varepsilon \) skilled workers can move and be at least weakly better off if they still receive \( w_{h,2} \), and this implies a firm that moves to the primary city is able to hire these \( \varepsilon \) skilled workers at the same wage rate before the move and increase profit by \( (A_1 - A_2) \left( \frac{\phi_t}{\phi_h} \right)^{1-\psi} \). Both cases lead to a contradiction. Suppose skilled workers are all in the primary city. Then \( \left( \frac{\bar{r}_1}{\bar{r}_2} \right)^\alpha = \left( \frac{\psi}{1-\psi} \right)^{\frac{\alpha}{1+\alpha}} \), as skilled and unskilled workers spend \( \alpha \psi \) and \( \alpha (1 - \psi) \) proportions of the economy’s output on housing in the primary and secondary cities, respectively. Thus, to keep skilled workers preferring the primary city, \( \left( \frac{A_1 \phi_t}{A_2 \phi_h} \right)^{\frac{1}{\psi}} \geq \left( \frac{\psi}{1-\psi} \right)^{\frac{\alpha}{1+\alpha}} \) must hold. But this is impossible given the parametric assumption. The last possibility now is that skilled workers are in both cities. This implies \( \left( \frac{\bar{r}_1}{\bar{r}_2} \right)^\alpha = \frac{w_{h,1}}{w_{h,2}} = \left( \frac{\bar{r}_1}{\bar{r}_2} \right)^\frac{1}{\psi} > 1 \). Thus, outsourcing must be from the primary to the secondary city, for otherwise \( \varepsilon \) unskilled workers
can be strictly better off by moving to the secondary city and being employed at the same wage rate before the move. ■

The above proposition assumes that $A_1 > A_2 e^{\gamma}$, which rules out multiple equilibria. This parametric assumption imposes that the primary city is always the more productive one regardless the skill shares of the two cities. Without this, there can exist low-level equilibria in which the secondary city’s producer is the one that outsources unskilled work. Nevertheless, the economy will not reach a low-level equilibrium if it starts off from no outsourcing to frictionless outsourcing, as the primary city initially has more costly unskilled workers and a higher rent. The proposition also assumes that $\left( \frac{A_1 e^{\gamma}}{2} \right)^{\frac{1}{\psi}} < \left( \frac{\psi}{1-\psi} \right)^{\frac{\alpha}{1+\alpha}}$, which rules out the possibility that skilled workers all live in one city. This exclusion is not worrisome, empirically. Without this, the model may not always predict an increase in $\frac{\rho_1}{\rho_2}$.

To recapitulate, this simple model can connect Facts 1, 2 and 3. It suggests that unskilled workers in the primary city, the more productive city, are more costly as they need to be compensated for the higher rent in this city. Thus, when technological progress eliminates outsourcing friction, the primary city’s producer will outsource unskilled work, and this makes the city more skilled and more productive relative to the secondary city. As the result, the rent increases relatively in the primary city, as the relative rent is determined by relative wage that is determined by relative productivity.

Despite its simplicity, this model has several limitations. First, it does not predict the diverging skill premiums between the two cities (Fact 5) that would support a theory with shifts in labor demand. Second, it is interesting to examine computerization (Fact 4) in addition to outsourcing, as the decline of computing prices is even more striking. Third, the previous analysis of the two extreme cases does not imply the impact is monotone. Furthermore, the parametric assumption in the proposition implies that $\psi$, the share of output of skilled workers in this simple model, is greater than one half. Although data do suggest that this share is increasing over time, it has not yet exceeded one half. To take all these into consideration, a richer model is introduced in the next section.
4 Richer Model

The richer model is build on the framework of the simple model, but a few modifications and additions are made here. Those elements not mentioned here are the same as in the previous section.

4.1 Preference

On the preference, workers weigh the two cities differently. The utility they can get from living in the primary city is:

$$u_{t,1} = \theta q_{t,1}^\alpha c_{t,1}^{1-\alpha}$$  \hspace{1cm} (9)

where $\theta$ denotes the value that the worker places on living in the primary city relative to the secondary city. Workers with a higher $\theta$ are associated with higher utility, ceteris paribus. This heterogeneity captures the notion that some workers have higher preference on living in the primary city while others have higher preference on the secondary city for exogenous reasons such as family connections. This assumption creates imperfect labor mobility and allows the two cities’ skill premiums to diverge. Other results do not depend upon this assumption$^{11}$. Assume that $\Theta$ follows a Pareto distribution, and let $F^c(\theta)$ denote $\Theta$’s complementary cumulative distribution function, i.e., $1 - F(\theta)$. We have

$$F^c(\theta) = \left(\frac{\theta}{\theta_0}\right)^\eta$$

where $\theta_0$ is the positive lower bound of the support. A nice property of this function is constant elasticity: A one percent increase in $\theta$ decreases $F^c(\theta)$ by $\eta$ percent. Additionally, it is assumed that $\Theta$ is identically distributed across the skill types. As for the utility of living in the secondary city, it is simply

$$u_{t,2} = q_{t,2}^\alpha c_{t,2}^{1-\alpha}$$  \hspace{1cm} (10)

$^{11}$Given the production technology specified later, complete outsourcing of unskilled work cannot occur in this richer model even without the assumption on $\theta$. 

4.2 Production

Assume two types of producers for the tradable numeraire. The "high-tech" producers can be present in both cities, and they employ both skilled and unskilled workers. For ease of presentation, their technology is separated into the production of final goods and intermediate goods, and it is assumed that each city has a representative final producer and a representative intermediate producer. The final producer has a CES technology:

\[ y_j = A_j \left( \psi h_j^{\sigma} + (1 - \psi) m_j^{\sigma} \right)^{\frac{1}{\sigma}} \]

where \( m_j \) is the amount of intermediate goods acquired in city \( j \), and \( \sigma \) is the elasticity of substitution.

The technology of the intermediate producer is crucial. The intermediate goods are non-tradable and have a local price \( \pi_j \). They are made by a Dixit-Stiglitz technology:

\[ m_j = \left( \int_0^1 \int_0^1 x_j (i_c, i_o)^{\mu-1} \, di_c di_o \right)^{\frac{\mu}{\mu-1}} \]

The intermediate goods are composed by a one-unit variety of differentiated tasks. In the literature, differentiated tasks are typically indexed along one dimension. However, this paper uses \( i_c \) and \( i_o \) to index these tasks in two dimensions, for its purpose. The subscript \( c \) indicates something about computers, and the subscript \( o \) indicates something about outsourcing. Details will be explained later. For each task, \( x_j \) is the quantity of the task used in producing the intermediate goods. The parameter \( \mu \) denotes the elasticity of substitution.

For each differentiated task, three production factors can be used, and they are perfect substitutes. They are local unskilled workers, computers, and remote unskilled workers in the other city (the outsourcing). Here, the central question is which factor should do which task. This is the first-stage problem of the intermediate producer. The producer determines
an optimal task assignment rule that assigns each task to the factor that can perform the task at the least cost. In the second stage, the producer takes the optimal task assignment rule as given and solves a standard cost minimization problem.

To determine which factor should do which task, the producer investigates the cost of producing one unit of the task for every task. This cost is called the unit cost in the remainder of the paper. Using local unskilled workers, the unit cost is

$$z_l = w_{l,j}$$

which equals the local unskilled wage, since it is assumed that one unskilled worker can do one unit of any task.

Using computers, the unit cost is

$$z_c(i_c; \zeta_c) = \zeta_c q_c(i_c)$$

The parameter $\zeta_c$ is the price of computers. (Recall the assumption of a small open economy. The supply is perfectly elastic, and therefore the price is fixed.) The function $q_c(i_c)$, which is called the computer requirement in later analysis, indicates the number of computers needed to complete one unit of the task indexed by $i_c$ and is independent of $i_o$. Assuming $q_c'(.) < 0$, $\lim_{i_c \to 0} q_c(i_c) = \infty$ and $\lim_{i_c \to 1} q_c(i_c) = 1$, there is heterogeneity in efficiency of computerization. Computers are bad at doing tasks indexed by small $i_c$, such as cleaning tables, whereas computers are good at doing tasks indexed by big $i_c$, such as audio recording. As $i_c = 1$, completing one unit of the task only requires one unit of computers.

Using outsourcing, the unit cost is

$$z_o(i_o; \zeta_o) = w_{l,-j} + \zeta_o q_o(i_o)$$

The intermediate producer needs to pay $\zeta_o q_o(i_o)$, the communication cost, in addition to, $w_{l,-j}$, the wage to an unskilled worker in the other city. The parameter $\zeta_o$ is the outsourcing
friction, which is the cost per unit of communication. The function $q_o(i_o)$, which is called the communication requirement in later analysis, indicates the amount of communication needed to outsource one unit of the task indexed by $i_o$ and is independent of $i_c$. Assuming $q'_o(.) < 0$, 
\[
\lim_{i_o \to 0} q_o(i_o) = \infty \text{ and } \lim_{i_o \to 1} q_o(i_o) = 0
\]
there is also heterogeneity in efficiency of outsourcing. Outsourcing tasks indexed by small $i_o$, such as the services of a personal secretary to a manager, needs substantial communication, whereas outsourcing tasks indexed by big $i_o$, such as mailing sales promotions, requires little communication. As $i_o = 1$, outsourcing is frictionless.

A "low-tech" producer in the secondary city can also make the tradable numeraire by using the city's unskilled workers as the sole input. The technology is constant return to scale and each of the unskilled workers can produce one unit of the numeraire. In the literature, it is not uncommon to assume two technologies for one single type of goods. Celebrated or often cited works that make this assumption include Murphy, Shleifer, and Vishny (1989a, 1989b) on industrialization, Yeaple (2005) on trade, and Beaudry and Green (2005) on labor economics. Here, the assumption of the low-technology in addition to the high-technology is to anchor the unskilled wage in the secondary city. This greatly simplifies analytical results, but the analysis will just focus on the case in which high- and low-technologies coexist in the secondary city. The model assumes that the less-skilled city can have a low technology but the skilled city cannot. This is in line with the emerging view in the literature that a greater supply of skilled workers in a locality can result in adoption of more advanced or skill-intensive technologies and make backward technologies being obsolete.

5 Equilibrium

This section and the rest of the paper focus on the interior equilibrium. Here, the main purposes are to derive the functions of primary city’s relative labor demand and relative labor supply, which are the two critical functions for later analysis, and to define the interior equilibrium.
5.1 How Things are Made

The high-tech final producers take the total factor productivity and factor prices as given and pick \( \{h_j, m_j\} \) to maximize profit. The solution is standard. The producer in city \( j \) demands

\[
\frac{m^D}{h_j} = \left( \frac{1 - \psi}{\psi} \right)^{\theta} \left( \frac{w_{h,j}}{\pi_j} \right)^{\theta}
\]  

(11)

units of intermediate goods per unit of skilled workers, and the skilled wage satisfies

\[
w_{h,j} = \psi^{\frac{\theta}{\sigma-\theta}} \left( A_j^{1-\sigma} - (1 - \psi)^{\sigma} \pi_j^{1-\sigma} \right)^{\frac{1}{\sigma}}
\]

In a special case where \( \gamma = 0 \), examined later, we have

\[
\frac{\partial w_{h,j}}{\partial \pi_j} = - \frac{m^D}{h_j}
\]  

(12)

because of the envelope theorem.

In each city, the high-tech intermediate producer solves a two-stage cost minimization problem. In the first stage, the producer determines an optimal task assignment rule. That is, the producer assigns each of the one-unit variety of differentiated tasks to the factor (local unskilled workers, remote unskilled workers or computers) that can produce the task at the least cost.

Figure 7 visualizes the optimal task assignment rules for the case where \( w_{l,1} > w_{l,2} > \zeta_c \). The interior equilibrium can only exist in this case. The left panel illustrates the rule of the primary city’s intermediate producer who has an incentive to outsource as unskilled workers are more expensive in this city. The producer also has an incentive to computerize because unskilled workers are more expensive than computers. As the result, tasks indexed by big \( i_o \) and small \( i_c \) will be outsourced. Conditional on outsourcing, the unit cost is \( z_o (i_o; \zeta_o) \). Similarly, tasks indexed by small \( i_o \) and big \( i_c \) will be computerized. Conditional on computerization, the unit cost is \( z_c (i_c; \zeta_c) \). As for tasks indexed by small \( i_c \) and small \( i_o \), both computerization and outsourcing are costly. Thus, these tasks will be performed in-house by local unskilled workers and the unit cost is \( w_{l,1} \).
In Figure 7, the optimal task assignment rule of the primary city's intermediate producer is characterized by three margins. The first margin is

$$i_{c,1} = q_c^{-1} \left( \frac{w_{l,1}}{\zeta_c} \right)$$

where $q_c^{-1}$ is the inverse function of $q_c$. On this margin, the producer is indifferent between using local unskilled workers and computers. The second margin is

$$i_{o,1} = q_o^{-1} \left( \frac{w_{l,1} - w_{l,2}}{\zeta_o} \right)$$

where $q_o^{-1}$ is the inverse function of $q_o$. On this margin, the producer is indifferent between using local unskilled workers and remote unskilled workers. The third margin is

$$\omega_1 : [i_{o,1}, 1] \to \left[ i_{c,1}, q_c^{-1} \left( \frac{w_{l,2}}{\zeta_c} \right) \right]$$

such that

$$\omega_1 (i_o) = q_c^{-1} \left( \frac{w_{l,2}}{\zeta_c} + \frac{\zeta_o}{\zeta_c} q_o (i_o) \right)$$

This margin is an increasing function of $i_o$. On this margin, the producer is indifferent between outsourcing and computerization.

The right panel of Figure 7 illustrates the optimal task assignment rule of the secondary
city’s intermediate producer. The producer does not outsource any task because unskilled workers are cheaper in this city. Thus, the rule is characterized by a single margin

\[ \hat{z}_{i,2} = q_c^{-1} \left( \frac{w_{i,2}}{\xi_c} \right) \]
on which the producer is indifferent between using local unskilled workers and computers.

In the second stage, each intermediate producer takes its optimal task assignment rule and the unit costs as given and chooses \( x_j (. , .) \) to solve a standard cost minimization problem. The price of the intermediate goods in the secondary city is

\[ \pi_2 = \left( \int_{0}^{\hat{i}_{c,2}} w_{i,c}^{1-\mu} di_c + \int_{\hat{i}_{c,2}}^{1} z_c(i_c; \xi_c)^{1-\mu} di_c \right)^{\frac{1}{1-\mu}} \]
and the price in the primary city is

\[ \pi_1 = \left( \int_{0}^{\hat{i}_{o,1}} \int_{0}^{\hat{i}_{c,1}} w_{i,c}^{1-\mu} di_o di_c + \int_{\hat{i}_{o,1}}^{1} \int_{0}^{\omega_1(i_o)} z_o(i_o; \xi_o)^{1-\mu} di_c di_o + \int_{0}^{1} \int_{0}^{1} z_c(i_c; \xi_c)^{1-\mu} di_o di_c + \int_{0}^{\omega_1(1)} \int_{0}^{1} z_c(i_c; \xi_c)^{1-\mu} di_o di_c \right)^{\frac{1}{1-\mu}} \]
where \( \omega_1^{-1} \) is the inverse function of \( \omega_1 \).

**Lemma 1** \( \frac{\partial \pi_1}{\partial \xi_o} > 0, \frac{\partial \pi_1}{\partial \xi_c} > 0, \frac{\partial \pi_2}{\partial \xi_c} > 0 \)

**Proof** In the appendix

This lemma is intuitive. Holding fixed \( w_{i,1} \), a lower \( \xi_o \) can decrease \( \pi_1 \) the price of primary city’s intermediate goods, because it lowers the unit costs of outsourced tasks. Similarly, a lower \( \xi_c \) can also decrease \( \pi_1 \) and \( \pi_2 \). In the rest of the paper, cost savings on producing intermediate goods due to a lower \( \xi_o \) or \( \xi_c \) will be referred to as *technological gain*.

The intermediate producers demand

\[ x_j = \left( \frac{\pi_j}{w_{i,j}} \right)^{\mu} m_j \]
units of local unskilled workers for each unskilled task that is neither outsourced nor computerized. Since the primary city’s producer assigns an \( i_{c,1} \) variety of tasks to local unskilled workers, it demands

\[
\frac{l^D}{m_1} = \hat{i}_{c,1} i_{o,1} \left( \frac{\pi_1}{w_{l,1}} \right)^\mu
\]  

(14)

units of local unskilled workers per unit of intermediate goods. Then, the multiplication of (11) and (14) is the primary city’s relative labor demand, i.e., the demand of unskilled workers relative to the demand of skilled workers:

\[
\frac{L^D}{H_1} = \hat{i}_{c,1} i_{o,1} \left( \frac{1 - \psi}{\psi} \right)^\sigma \left( \frac{\pi_1}{w_{l,1}} \right)^\mu \left( \frac{w_{h,1}}{\pi_1} \right)^\sigma
\]  

(15)

Additionally, for later use, note that

\[
\frac{\partial \pi_1}{\partial w_{l,1}} = \frac{l^D}{m_1}
\]  

(16)

by the envelope theorem. As for the secondary city, it can be noted that an

\[
\hat{i}_{c,2} \left( \frac{1 - \psi}{\psi} \right)^\sigma \left( \frac{\pi_2}{w_{l,2}} \right)^\mu \left( \frac{w_{h,2}}{\pi_2} \right)^\sigma H_2 + \int_{i_{o,1}}^{1} \int_{i_{o,1}}^{\omega_1(i_o)} z_o(i_o; \zeta_o)^{-\mu} di_o d\mu \pi_{1}^{\mu} \left( \frac{1 - \psi}{\psi} \right)^\sigma \left( \frac{w_{h,1}}{\pi_1} \right)^\sigma H_1
\]

units of local unskilled workers are demanded by high-tech producers, where the first term is the demand from the local high-tech producer and the second term is the demand from the primary city’s high-tech producer.

### 5.2 Who Lives Where

Workers can choose where to live, but they must not have an incentive to relocate in equilibrium. Thus, the indirect utility of those workers living in the primary city must be no less than the indirect utility they could get if they move to the secondary city, and vice versa. This implies a cutoff value for each type of workers:

\[
\hat{\theta}_t = \left( \frac{w_{t,2}}{w_{t,1}} \right)^\alpha \left( \frac{r_1}{r_2} \right)^\alpha
\]  

(17)
Type $t$ workers live in the primary city if and only if their $\theta$ value is greater than $\hat{\theta}_t$.

Given the cutoff values, the primary city’s supplies of skilled and unskilled workers, respectively, are

$$H^S_i = \phi_h \left( \frac{\theta}{\theta_h} \right)^\eta$$

and

$$L^S_i = \phi_l \left( \frac{\theta}{\theta_l} \right)^\eta$$

Through the assumption on $F^c(\cdot)$, these two supply functions have a nice property — constant elasticity. The rent elasticity is $-\alpha \eta$, and the wage elasticity is $\eta$. In addition, $\eta$ has an implication on labor mobility. As $\eta \to 0$, the probability density $f(\theta) \to 0$ for every $\theta$. Thus, varying the cutoff value only makes measure zero of workers relocate; there is no mobility. As $\eta \to \infty$, workers identically value the primary city, so they are perfectly mobile. Generally speaking, a bigger $\eta$ implies that a greater proportion of workers will relocate when wages or rent changes. It is noteworthy that this imperfect labor mobility is due to heterogeneous preference but not relocation costs.

Given (18) and (19), the primary city’s relative labor supply, i.e., the supply of unskilled workers relative to the supply of skilled workers, is

$$\frac{L^S_i}{H^S_i} = \frac{\phi_l}{\phi_h} \left( \frac{w_{l,1} w_{h,2}}{w_{l,2} w_{h,1}} \right)^\eta$$

This function is independent of rent, because the two types of workers are equally mobile.

Lastly, the individual housing demand is

$$q_{t,j} = \frac{\alpha w_{t,j}}{r_j}$$

which is derived from the utility maximization, and the housing demand of city $j$ is

$$Q^D_j = q_{h,j} H_j + q_{l,j} L_j$$
which is the aggregate of the individual demand.

**Definition 1** The interior equilibrium consists of a vector of prices—$(w_{l,1}, w_{h,1}, w_{h,2}, r_1, r_2, \pi_1, \pi_2)$, a vector of cutoff values—$(\hat{\theta}_h, \hat{\theta}_l)$, task assignment rules—$(\hat{i}_{c,1}, \hat{i}_{o,1}, \omega_1(.))$ and $\hat{i}_{c,2}$, and consumption and factor allocations such that the following conditions hold:

1. Workers’ decisions, including the location choices, maximize utility.
2. Producers’ decisions, including the task assignment, maximize profit.
3. Labor markets clear.
4. Housing markets clear.
5. The two cities are inhabited by both skilled and unskilled workers.
6. In the secondary city, the low-tech producer exists and employs a positive amount of local unskilled workers at the wage rate $w_{l,2} = 1$.
7. $\hat{i}_{c,1}$, $\hat{i}_{o,1}$, and $\hat{i}_{c,2}$ are all less than one.

This paper focuses on the interior equilibrium, which exists and is unique within a plausible range of parameter values as shown in a later section. In general, $\phi_l$ has to be sufficiently big so that some unskilled workers will be employed by the low-tech producer in equilibrium, $A_l$ has to be sufficiently big relative to $A_h$ so that the primary city’s unskilled wage will be higher and outsourcing will occur, and $\hat{\theta}$ has to be sufficiently small so that the secondary city will be resided with both types of workers.

### 6 Impact of Outsourcing and Computerization

This section shows that both lower outsourcing friction (lower $\zeta_o$) and lower computer price (lower $\zeta_c$) are able to increase disparities in the skill share, skill premiums and rent between the two cities, although stronger assumptions are needed for the case of lower computer price. The discussion starts with the impact of outsourcing. This is a simpler case, because lower outsourcing friction does not affect wages in the secondary city. The section focuses on the
case $\gamma = 0$ in which the TFP is independent of the skill share. In the next section, it is shown computationally that the impact is bigger when $\gamma > 0$.

In this section, the most important equations are (15) and (20)—the primary city’s relative labor demand and relative labor supply, which are functions of $w_{l,1}$, as $w_{h,1}$ and $\pi_1$ are functions of $w_{l,1}$ as well. If lower $\zeta_o$ and lower $\zeta_c$ can both decrease the two functions holding fixed $w_{l,1}$, then the theory can be validated without much difficulty. The proposition below gives the condition under which the relative labor demand decreases.

**Proposition 2** If $\sigma$ is sufficiently small, there is technology-skill complementarity. That is, the primary city’s relative labor demand decreases when $\zeta_o$ or $\zeta_c$ decreases.

**Proof** In the appendix

The intuition of this proposition runs as follows. Recall the primary city’s relative labor demand is the multiplication of (11) and (14). Holding fixed $w_{l,1}$, a lower $\zeta_o$ ($\zeta_c$) affects this relative labor demand through two channels. First, the lower $\zeta_o$ ($\zeta_c$) not only triggers outsourcing (computerization) and decreases the number of differentiated tasks being performed by local unskilled workers in the primary city, but also results in the substitution of outsourced (computerized) tasks for in-house tasks. This leads to Fact 3, because, given unskilled wage is higher in the primary city, the lower $\zeta_o$ only incurs outsourcing from the primary to the secondary city (the lower $\zeta_c$ induces more computerization in the primary city). Moreover, this decreases (14) the intermediate producer’s demand for local unskilled workers per unit of intermediate goods. As the result, the impact of the lower $\zeta_o$ ($\zeta_c$) through this first channel tends to decrease the primary city’s relative labor demand. On the other hand, the impact through the second channel tends to increase the relative labor demand, as the lower $\zeta_o$ ($\zeta_c$) increases (11) the final producer’s demand for intermediate goods per unit of skilled workers; the lower $\zeta_o$ ($\zeta_c$) decreases the intermediate goods price, which in turn leads to the substitution of intermediate goods for skilled workers. The value of $\sigma$ determines the increment of (11). The lower is $\sigma$, the smaller is the increment. On an extreme that $\sigma = 0$, the final producer’s technology is Leontief and its demand for intermediate goods per unit of skilled workers is
constant. Thus, when $\sigma$ is sufficiently small, the impact through the first channel dominates, and the lower $\zeta_o$ ($\zeta_o$) decreases the primary city’s relative labor demand. For the rest of the paper, the following assumption is made.

**Assumption 1** The parameter $\sigma$ is sufficiently small such that there is technology-skill complementarity.

### 6.1 Impact of Outsourcing

**Proposition 3** A lower $\zeta_o$ increases the skill share in the primary city but decreases it in the secondary city.

**Proof** In the appendix

With the technology-skill complementarity, lower outsourcing friction can decrease the primary city’s relative labor demand (15) holding fixed $w_{l,1}$. Additionally, the primary city’s relative labor supply (20) also decreases for the following reason. The lower friction decreases the price of intermediate goods in the primary city (Lemma 1), and this technological gain permits skilled workers to earn a higher wage there ($\frac{\partial w_{h,1}}{\partial \sigma_1} = -\frac{m^D}{h_1} < 0$). As the result, a greater number of skilled workers choose living in the primary city; the relative labor supply decreases. Since the demand and supply both decrease, the number of unskilled workers relative to the number of skilled workers in the primary city decreases. This implies that the skill share increases in the primary city and decreases in the secondary city. The above intuition for Proposition 3 also serves as an explanation for Fact 2, although the proposition does not predict a higher skill share in the secondary city as the model abstracts away from rising $\phi_h$, the proportion of workers who are skilled in the entire economy.

**Proposition 4** A lower $\zeta_o$ increases the skill premium in the primary city relative to the premium in the secondary city.

**Proof** In the appendix
This proposition is about Fact 5. As mentioned earlier, the primary city’s relative labor supply is independent of \( r_1 \) and \( r_2 \), because the supply functions of the two types of workers have the same elasticity with respect to the rents. Thus, by (20), a decrease in the relative number of unskilled to skilled workers in the primary city implies that the skill premium in the primary city increases relative to the premium in the secondary city.

**Proposition 5** A lower \( \zeta_o \) increases \( w_{h,1} \), but its effect on \( w_{l,1} \) is ambiguous. However, if \( \eta \) is sufficiently big, then \( w_{l,1} \) increases.

**Proof** In the appendix

This proposition is used as a stepping stone to the result on rent. To see why this proposition is true, first note that the impact of lower outsourcing friction on the primary city’s skilled wage can be written as

\[
\frac{dw_{h,1}}{d\zeta_o} = \frac{dw_{h,1}}{d\pi_1} \left( \frac{\partial \pi_1}{\partial \zeta_o} + \frac{\partial \pi_1}{\partial w_{l,1}} \frac{dw_{l,1}}{d\zeta_o} \right) \tag{21}
\]

Clearly, if a lower \( \zeta_o \) decreases \( w_{l,1} \), then \( w_{h,1} \) will increase, because the technological gain and the lower \( w_{l,1} \) both decrease the price of intermediate goods and therefore increase the marginal productivity of skilled workers. Nevertheless, \( w_{h,1} \) will still increase even if a lower \( \zeta_o \) increases \( w_{l,1} \). This is implied by Proposition 4 and (20).

The impact of a lower \( \zeta_o \) on \( w_{l,1} \) is ambiguous. On the one hand, decrease in the relative labor demand tends to push down \( w_{l,1} \). On the other hand, decrease in the relative labor supply tends to drive the wage up. Whether \( w_{l,1} \) eventually increases depends upon \( \eta \), which can be interpreted as the degree of labor mobility as it is the wage elasticity of the two labor supply functions–(18) and (19)–of the primary city. If \( \eta \) is small, the increase in \( w_{h,1} \) has a minimal effect on the supply of skilled workers and consequently on the relative labor supply. Then, the impact of the decrease in the relative labor demand would dominate and \( w_{l,1} \) would decrease. On the contrary, if \( \eta \) is big, the increase in \( w_{h,1} \) can attract a large number of skilled workers from the secondary to the primary city. Then, the decrease in the relative labor supply can be substantial. As long as the impact of this relative labor supply shift on \( w_{l,1} \) dominates, \( w_{l,1} \) can increase.
For concreteness, let us consider two examples. In the first example, \( \eta \to 0 \). The lower \( \zeta_\eta \) leads to zero migration and does not shift the relative labor supply, because the mass of \( \theta \) around the cutoff value converges to zero. In this case, \( w_{l,1} \) decreases unambiguously. In the second example, \( \eta \to \infty \). This implies that workers value the primary city identically and we can normalize \( \theta \) to 1 for every worker. This is the case of perfect labor mobility commonly seen in the literature, and (17) implies that:

\[
\frac{w_{h,1}}{w_{h,2}} = \frac{w_{l,1}}{w_{l,2}}
\]

Since the lower \( \zeta_\eta \) increases \( w_{h,1} \), but \( w_{h,2} \) and \( w_{l,2} \) remain unchanged, \( w_{l,1} \) must increase.

**Proposition 6** When \( \zeta_\eta \) decreases, the rent in the primary city increases relative to the rent in the secondary city if \( \eta \) is sufficiently big.

**Proof** In the appendix

This proposition is related to Fact 1 and the intuition is the following. If \( \eta \), the labor mobility, is sufficiently big such that the lower \( \zeta_\eta \) increases both skilled and unskilled wages in the primary city, then the city’s supply functions of skilled and unskilled workers both increase holding fixed rents. This “migration effect” increases housing demand in the primary city and decreases the demand in the secondary city. Thus, the relative rent, \( \frac{r_1}{r_2} \), can increase. Moreover, the higher wages in the primary city generate an additional positive income effect that also pushes up the relative rent.

The parametric restriction on \( \eta \) respects data, indeed. Empirical literature suggests that labor is mobile. For instance, the estimate of Gallin (2004) implies that the labor mobility, \( \eta \), is between 1.5 and 2. (See Rappaport, 2005 for a discussion on this implication.) Nonetheless, if labor is immobile, a lower \( \zeta_\eta \) may decrease the relative rent under certain parameter values. Figure 8 illustrates such an example.
Figure 8: Lower $\zeta_o$ may decrease relative rent if $\eta$ is small.

6.2 Impact of Computerization

The impact of lower computer price is more complex, because the lower price also affects the skilled wage in the secondary city. To develop the analysis, I first present the next proposition.

Proposition 7 *The number of computers per worker is bigger in the primary city.*

Proof In the appendix

To see why this proposition, which is on Fact 4, is true, it is most important to note that the primary city’s intermediate producer uses more computers and fewer local unskilled workers in producing every unit of intermediate goods as compared to its counterpart in the secondary city. This is because unskilled workers are more expensive in the primary city, and, therefore, the primary city’s intermediate producer not only assigns a greater variety of the differentiated tasks to computers and a smaller variety to local unskilled workers, but also is more inclined to use computerized tasks to substitute for those tasks performed by local unskilled workers.

Corollary 1 $\frac{\partial}{\partial c_c} \left( \frac{\pi_1}{\pi_2} \right) > 0$, holding fixed the unskilled wage in the primary city.

Proof In the appendix
This corollary follows directly from Proposition 7. When the computer price decreases, the intermediate goods in the primary city will become cheaper relative to the goods in the secondary city, because the primary city’s intermediate producer uses computers more extensively and more intensively in its production.

Assumption 2 \( \frac{\partial}{\partial \zeta_c} \left( \frac{w_{h,1}}{w_{h,2}} \right) < 0 \)

Assumption 2 ensures that lowering the computer price will decrease the primary city’s relative labor supply holding fixed the city’s unskilled wage, as (20) has to hold. Given Assumption 1 on the technology-skill complementarity, the lower computer price will decrease the primary city’s relative labor demand. Thus, if Assumption 2 is also imposed, the previous analysis on the impact of lower outsourcing friction can apply here.

From the empirical perspective, Assumption 2 is in order, since research finds that the wage premium for skilled workers to work in skilled cities has been increasing over past decades (e.g., Berry and Glaeser, 2005), while computer prices have been decreasing. From the theoretical perspective, Assumption 2 is needed, because the lower \( \zeta_c \) can only increase \( \frac{w_{h,1}}{w_{h,2}} \) within a range of parameter values. More specifically, whether \( \frac{\partial}{\partial \zeta_c} \left( \frac{w_{h,1}}{w_{h,2}} \right) \) is negative or positive depends upon the complementarity between skilled workers and intermediate goods and the productivity difference between the two cities. To see this, let us consider two examples. In the first example, \( \sigma = 1 \). The production function of the final producers is Cobb-Douglas, and the wage premium for skilled workers to work in the primary city can be written as

\[
\frac{w_{h,1}}{w_{h,2}} = \left( \frac{A_1}{A_2} \right)^{\frac{1}{\sigma}} \left( \frac{\pi_2}{\pi_1} \right)^{\frac{1-\alpha}{\alpha}}
\]

Given Corollary 1, the lower \( \zeta_c \) tends to increase this wage premium without ambiguity. In the second example, in which \( \sigma = 0 \), skilled workers and intermediate goods are perfect complements and the wage premium for skilled workers to work in the primary city can be written as

\[
\frac{w_{h,1}}{w_{h,2}} = \frac{A_1 - \pi_1}{A_2 - \pi_2}
\]
Clearly, if $A_1$ is big relative to $A_2$, the lower $\zeta_c$ may decrease this wage premium. If this is the case, then the primary city’s relative labor supply will increase and my theory may not hold. Therefore, Assumption 2 is made for the rest of the paper and the parameter values being considered must respect this assumption.

**Proposition 8** A lower $\zeta_c$ has the following impacts:

1. The skill share increases in the primary city while it decreases in the secondary city.
2. The skill premium in the primary city increases relative to the premium in the secondary city.
3. A lower $\zeta_c$ increases $w_{h,1}$ and $w_{h,2}$, but its impact on $w_{l,1}$ is ambiguous. However, if $\eta$ is sufficiently big, then $w_{l,1}$ increases.
4. The rent in the primary city increases relative to the rent in the secondary city if $\eta$ is sufficiently big.

**Proof.** In the appendix.

This proposition is consistent with the various stylized facts presented in Section 2. Moreover, it is in line with the correlation measures between computer prices and the stylized facts. To see why this proposition is true, most of the intuitions discussed in Section 6.1 can be applied here, since the lower $\zeta_c$ also results in decreases in both the relative demand and supply of labor given Assumptions 1 and 2. But point 4, the result on rent, needs more explanation. On the one hand, the lower $\zeta_c$ affects wages and leads to migration of skilled and unskilled workers to the primary city which tends to increase the relative rent, $\frac{p_1}{p_2}$. On the other hand, the lower $\zeta_c$ does increase $w_{h,2}$ which generates a positive income effect on housing consumption for secondary city’s skilled workers, and this tends to decrease the relative rent. Overall, as long as $\eta$ is sufficiently big such that the lower $\zeta_c$ can lead to an adequate amount of migration to the primary city, the relative rent will increase.

This section analyzes the impacts of lower outsourcing friction and computer price in the special case that $\gamma = 0$. The next section numerically shows that these impacts increase in the value of $\gamma$. It also shows how the impacts depend upon values of various parameters and discusses the intuitions behind these relationships.
7 Simulation

This section presents numerical comparative statics, which shed light on how the impacts of outsourcing and computerization depend upon parameter values; it does not mean to be an exact quantitative assessment. To facilitate this numerical exercise, the functions of communication and computer requirement are assumed to be

\[ q_o (i_o) = i_o^{\rho_o} \]

and

\[ q_c (i_c) = i_c^{\rho_c} \]

respectively, where \( \rho_o \) and \( \rho_c \) are shape parameters.

A benchmark of parameters is chosen and is summarized in Table 2. Then, parameters are perturbed one at a time, to see how sensitive the effects of lower \( \zeta_o \) and \( \zeta_c \) are to the parameter values. The benchmark value of the substitution elasticity between differentiated tasks—\( \mu \)—is set to 1.4, and the substitution elasticity between high and low skills—\( \sigma \)—is set to 0.7. These values are well within the range of the estimates reviewed by Hamermesh (1993).

Table 2: Summary of Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark value</th>
<th>Source or target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>1.4</td>
<td>Hamermesh (1993)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.7</td>
<td>Hamermesh (1993)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.3</td>
<td>Acemoglu (2002)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.4</td>
<td>Moretti (2004) and others</td>
</tr>
<tr>
<td>( \Delta_1 )</td>
<td>2.5</td>
<td>Wage premiums of skilled cities</td>
</tr>
<tr>
<td>( \Delta_2 )</td>
<td>0.9*( \Delta_1 )</td>
<td>Wage premiums of skilled cities</td>
</tr>
<tr>
<td>( \rho_o )</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>( \zeta_o )</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>( \zeta_c )</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>1.75</td>
<td>Gallin (2004)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.67</td>
<td>( \theta=1 ) at the 50\textsuperscript{th} percentile of ( \theta ) distribution</td>
</tr>
<tr>
<td>( \phi_h )</td>
<td>0.3</td>
<td>Share of urban workers who are skilled</td>
</tr>
<tr>
<td>( \chi )</td>
<td>2</td>
<td>Green et al. (2005)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.24</td>
<td>Morris and Ortalo-Magane (2008)</td>
</tr>
</tbody>
</table>
The parameter $\psi$ is set to 0.3. Acemoglu (2002) studies skill premiums from 1940 to 1990 and finds that the term of factor-augmenting technology of the high skill relative to the term of the low skill had been increasing. The relative value $\left(\frac{\psi}{1-\psi}\right)$ was 0.47 in 1990. This implies that $\psi$ was near 0.32. Thus, 0.3 is chosen as the benchmark value for $\psi$.

The parameter on human capital spillovers--$\gamma$--is set to 0.4. The empirical literature shows mixed evidences for this source of externality. The results of Rauch (1993) and Moretti (2004) are positive, while the results of Acemoglu and Angrist (2000) and Ciccone and Peri (2006) are conservative. This may be an area that needs further research, as it is arguable whether research has succeeded in addressing selection and omitted variables. Thus, 0.4, a medium level of $\gamma$, is chosen as the benchmark value, and then the impacts of lower $\zeta_o$ and $\zeta_c$ are studied for a wide $\gamma$ interval $[0, 0.6]$ where the upper bound is Moretti’s (2004) estimate for $\gamma$. Additionally, $A_1$ is set to 2.5 and $A_2$ is ninety percent of $A_1$. These two numbers are chosen to roughly match the skilled and unskilled workers’ wage premiums in skilled cities, where skilled cities are those with a skill share greater than the nation’s share.

The two parameters $\rho_o$ and $\rho_c$ in the functions of communication and computer requirement will be critical, if the goal of the simulation was to identify exact quantitative impacts. For example, if $\rho_c$ is small, then the 90% decrease in computing price over past two decades would completely change the world; we would see robots cleaning tables in restaurants. However, as the weight of computer services in U.S. GDP only increased from 0.4% to 1.6% over the period (Jorgenson, 2001), the value of $\rho_c$ may be big. I am not aware of a research work that estimates $\rho_o$ or $\rho_c$. Thus, I first set $\rho_o = 2$ and $\rho_c = 2$ as the benchmark values and then study the impacts of lower $\zeta_o$ and $\zeta_c$ for wide $\rho_o$ and $\rho_c$ intervals. In addition, the initial levels of $\zeta_o$ and $\zeta_c$ are both set to 0.5.

Although perfect labor mobility is a common assumption in theoretical literature, estimates obtained from empirical research such as Barro et al. (1991) and Greenwood, et al. (1991) are not as high as one would expect. However, the estimate of Gallin (2004) implies a higher mobility, one that is between 1.5 and 2 (Rappaport, 2005, discusses this implication). Thus, I set the benchmark value of labor mobility $\eta$ to 1.75 and study the impacts for a wide $\eta$ interval.
In addition, $\theta$ is set to 0.67 so that the worker who stands at the 50th percentile of $F(\theta)$ has $\theta = 1$, and $\phi_h$ is set to 0.3 which roughly matches current share of urban workers who are skilled.

The housing supply elasticity $\chi$ is set to 2. Green et al. (2005) review the national level estimates of the long run elasticity of housing supply. The estimates are between 1.5 and 4. They further estimate the elasticity for 45 U.S. metropolitan areas. The estimates range between 0 and 30. Thus, the benchmark value of $\chi$ is set to 2 and the impacts are studied for a wide $\chi$ interval. In addition, the income share of housing consumption--$\alpha$--is set to 0.24 as suggested in Morris and Ortalo-Magne (2008).

In the range of parameter values tested below, the equilibrium is unique and interior. In addition, the equilibrium wages and rent are consistent with U.S. patterns: (i) the skill premium is larger in the primary (skilled) city, (ii) both skilled and unskilled workers in the primary (skilled) city have a wage premium compared with those in the secondary (less-skilled) city, and (iii) rent is higher in the primary (skilled) city.

The discussion below focuses on how the impacts of lower $\zeta_o$ and $\zeta_c$ depend upon $\gamma$, $\rho_o$, $\rho_c$, $\eta$ and $\chi$ these five parameters, as the impacts are less sensitive to other parameters. All the presented figures illustrate the effects of a one percent decrease in outsourcing friction $\zeta_o$ or in computer price $\zeta_c$ by the parameter chosen. As shown in the legends, the three dash lines, respectively, are for (i) the relative rent which is the rent in the primary city relative to the rent in the secondary city, (ii) the skill share of the primary city, and (iii) the relative skill premium which is the skill premium in the primary city relative to the premium in the secondary city.

It is not surprising to see that the effects on the skill share and relative skill premium have proximate curves in most of the figures, as (20) must be satisfied in equilibrium.

Figures 9 and 10 illustrate the effects of a one percent decrease in $\zeta_o$ and $\zeta_c$, respectively, by $\gamma$ the parameter of human capital spillovers. The effects on the relative rent, skill share and relative skill premium all increase in the value of $\gamma$. The reason is that the shifts in the relative labor demand and supply result in divergence of the skill shares which in turn enlarges the productivity difference between the two cities and strengthen the effects.
The impact of the lower computer price is much stronger than the impact of the lower outsourcing friction. When $\gamma$ is 0.6, a one percent decrease in $\zeta_c$ increases the relative rent, skill share and relative skill premium by about 2%, while the same percentage decrease in $\zeta_o$ only increases them by about 0.2%. Nevertheless, as prices of communications fell by about 40% just in the second half of the 1990s (Doms, 2005), the impact of this change deserves consideration.

Figures 11 and 12 illustrate the effects of a one percent decrease in $\zeta_o$ and $\zeta_c$ by $\rho_o$ and $\rho_c$, respectively. The effects are sensitive to and decreasing in parameter value, for the reason mentioned earlier in this section. For instance, the relative rent, skill share and relative skill premium will increase by around 2.5% if $\rho_c$ is equal to 1.5 but only 0.15% if $\rho_c$ is equal to 4.5.
Nevertheless, the impact of the 90% decrease in computer prices over past two decades would be still significant, even if $\rho_c$ is big.

Figure 11: $\rho_o$ comparative statics given a 1% decrease in $\zeta_o$

![Figure 11](image1.png)

Figure 12: $\rho_c$ comparative statics given a 1% decrease in $\zeta_c$

![Figure 12](image2.png)

Figures 13 and 14 illustrate the effects by labor mobility $\eta$. The percentage increase in the relative skill premium is small when $\eta$ is big and converges to zero when $\eta$ converges to infinity, for the reason explained in Section 6.1. On the other hand, the percentage increases in the relative rent and skill share increase in $\eta$. If $\eta$ is zero, then workers do not move and the lower $\zeta_o$ and $\zeta_c$ have no impact on the skill share. However, there is still a positive impact on the relative rent due to the income effect on housing consumption, but the impact is quite minimal, as the income effect itself does not alter the levels of human capital spillovers. Thus, it is the
migration effect and the consequent change in human capital spillovers that make the impact big.

Figure 13: $\eta$ comparative statics given a 1% decrease in $\zeta_o$

![Graph](image1.png)

Figure 14: $\eta$ comparative statics given a 1% decrease in $\zeta_e$

![Graph](image2.png)

Figures 15 and 16 illustrate the effects by housing supply elasticity $\chi$. Clearly, the impact on the relative rent decreases in $\chi$. On the contrary, the effects on the skill share and relative skill premium increase in $\chi$, because a more elastic supply of housing allows a larger number of skilled workers to live in the primary city, and a higher skill share in the primary city implies a bigger relative skill premium by equation (20).
8 Conclusion

Using a model with technology-skill complementarity and heterogeneous productivity, this paper analyzes how outsourcing and computerization could affect spatial allocation of production activities and impact the cities’ labor and housing markets. The paper documents five stylized facts: (i) Housing prices increase faster in skilled cities. (ii) Skill share increases faster in skilled cities. (iii) Unskilled business support jobs are increasingly concentrated in less-skilled cities. (iv) Skilled cities use computers more intensively. (v) Skill premium increases faster in skilled cities. Furthermore, the research identifies a correlation between the emergence of these facts
and computer prices, although the correlation check is not feasible for prices of communications due to data constraint. The model’s ability to simultaneously capture these five stylized facts with a decrease in prices of either computers or communications suggests that computerization and domestic outsourcing may be good explanations for the rise of skilled cities and thus deserve more research attention than they currently receive.

The paper’s simulation exercise suggests that human capital spillovers can enlarge the impacts of outsourcing and computerization. This externality might raise efficiency and policy questions. For instance, should governments encourage outsourcing? Outsourcing could lead to more efficient use of spaces, but would people in less-skilled cities left behind? Particularly, if neighborhood effects are important, being isolated from skilled people might have an adverse impact on less-skilled cities’ residents and their children. This deserves future research.

Additionally, future research may consider threshold externality. As mentioned, Facts 1 and 2, the correlations between initial skill share and later changes in housing prices and skill share seem to be largely affected by the most highly skilled cities. This might suggest existence of nonlinear, threshold effects. Future research could consider city growth models with threshold externality of human capital to further understand the rise of skilled cities.

This paper might also have an implication regarding the fact that unskilled immigrants increasingly move out of several U.S. metropolitan areas, that are skilled yet popular for international migrants, since rising housing prices might have made housing less affordable to unskilled immigrants and they would prefer moving to other cities where housing is cheaper and unskilled jobs are more available.

\section*{A Appendix}

\subsection*{A.1 Supplement to Section 3}

Here, more details on equilibrium characterization are included. First, if outsourcing is not possible, then

\[(H_1, H_2, L_1, L_2) = (h_1, h_2, l_1, l_2)\]
and
\[
(h_1, h_2, l_1, l_2) = \left( \left( \frac{A_1}{A_2} \right)^{\frac{\psi+1}{\psi \alpha}} + 1 \right)^{-1} \left( \frac{A_1}{A_2} \right)^{\frac{\psi+1}{\psi \alpha}} \phi_h, \phi_h, \left( \frac{A_1}{A_2} \right)^{\frac{\psi+1}{\psi \alpha}} \phi_l, \phi_l \right)
\]

The equilibrium wages and rents are in turn determined by the above allocation.

Second, if outsourcing is frictionless and \( e^r < \frac{A_1}{A_2} < \left( \frac{\psi}{1-\psi} \right)^{\frac{\psi+1}{\psi \alpha}} e^{-\gamma} \), then
\[
(H_1, H_2, L_1, L_2) = (h_1, h_2, 0, \phi_l)
\]

and
\[
(h_1, h_2, l_1, l_2) = \left( \frac{A_1}{A_2} \right)^{\frac{\psi+1}{\psi \alpha}} \phi_l, \phi_l, \left( \frac{A_1}{A_2} \right)^{\frac{\psi+1}{\psi \alpha}} \phi_h, \phi_h \right)
\]

where
\[
(A_1, A_2) = (A_1 e^r, A_2 e^{\gamma \frac{H_2}{z_2+\gamma}})
\]

The equilibrium wages and rents are in turn determined by the above allocation.

### A.2 Supplement to Sections 5 and 6

**Proof of Lemma 1.** To see \( \frac{\partial \pi_1}{\partial \zeta_c} > 0 \). Equation (13) can be rewritten as
\[
\pi_1 = \left( i_o, i_c, w_{1,1}^{-\mu} + i_{i,c,1} \int_{i_o,1}^{i_{i,c,1}} z_o (i_o; \zeta_o)^{-\mu} d_i + \int_{i_{i,c,1}}^{\omega_1(1)} \int_{i_o,1}^{1 \omega_1(1)} z_o (i_o; \zeta_o)^{-\mu} d_i d_i + \int_{i_{i,c,1}}^{\omega_1(1)} \omega_1^{-1}(i_c) z_c (i_c; \zeta_c)^{-\mu} d_c + \int_{i_{i,c,1}}^{1 \omega_1(1)} z_c(i_c; \zeta_c)^{-\mu} d_c \right)^{-\frac{1}{\mu}}
\]

Taking derivative with respect to \( \zeta_c \), holding fixed \( w_{1,1} \), applying Leibniz’s Formula along the calculation, and using the fact that \( z_o(\omega_1^{-1}(i_c); \zeta_o) = z_c(i_c; \zeta_c) \), we have
\[
\frac{\partial \pi_1}{\partial \zeta_c} = \pi_1^{\mu} \left( \int_{i_{i,c,1}}^{\omega_1(1)} \omega_1^{-1}(i_c) z_c(i_c; \zeta_c)^{-\mu} q_c(i_c) d_i + \int_{\omega_1(1)}^{1} z_c(i_c; \zeta_c)^{-\mu} q_c(i_c) d_i \right) > 0
\]
Similarly, we have

\[ \frac{\partial \pi_1}{\partial \zeta_o} = \pi_1^{\mu} \left( \int_{i_o,1}^1 \omega_1(i_o) z_o(i_o; \zeta_o)^{-\mu} q_o(i_o) \, di_o \right) > 0 \]

and

\[ \frac{\partial \pi_2}{\partial \zeta_c} = \pi_2^{\mu} \left( \int_{i_c,2}^1 z_c(i_c; \zeta_c)^{-\mu} q_c(i_c) \, di_c \right) > 0 \]

Proof of Proposition 2. Taking the derivative of \( \frac{L^D}{\pi_1} \) with respect to \( \zeta_o \), we have

\[ \frac{\partial \frac{L^D}{\pi_1}}{\partial \zeta_o} = \frac{L^D}{H_1} \left( \frac{1}{i_o,1} \frac{\partial i_o,1}{\partial \zeta_o} + \frac{\mu}{\pi_1} \frac{\partial \pi_1}{\partial \zeta_o} \right) - \sigma \left( \frac{m^D}{h_1} \frac{1}{w_{h,1}} + \frac{1}{\pi_1} \right) \frac{\partial \pi_1}{\partial \zeta_o} \]

Similarly,

\[ \frac{\partial \frac{L^D}{\pi_1}}{\partial \zeta_c} = \frac{L^D}{H_1} \left( \frac{1}{i_c,1} \frac{\partial i_c,1}{\partial \zeta_c} + \frac{\mu}{\pi_1} \frac{\partial \pi_1}{\partial \zeta_c} \right) - \sigma \left( \frac{m^D}{h_1} \frac{1}{w_{h,1}} + \frac{1}{\pi_1} \right) \frac{\partial \pi_1}{\partial \zeta_c} \]

Since \( \frac{\partial i_o,1}{\partial \zeta_o} > 0 \), \( \frac{\partial i_c,1}{\partial \zeta_c} > 0 \), \( \frac{\partial \pi_1}{\partial \zeta_o} > 0 \), and \( \frac{\partial \pi_1}{\partial \zeta_c} > 0 \), we have \( \frac{\partial \frac{L^D}{\pi_1}}{\partial \zeta_o} > 0 \) and \( \frac{\partial \frac{L^D}{\pi_1}}{\partial \zeta_c} > 0 \) as long as \( \sigma \) is sufficiently small. ■

Proof of Proposition 3. Since \( \frac{\partial L^S}{\partial \zeta_o} > 0 \) and

\[ \frac{\partial \frac{L^S}{H_1}}{\partial \zeta_o} = \frac{L_1}{H_1 w_{h,1}} \frac{M_1}{H_1} \frac{\partial \pi_1}{\partial \zeta_o} > 0 \]

(by equation 12 and Lemma 1), lower \( \zeta_o \) must lead to lower \( \frac{L^S}{H_1} \). Thus, the primary city’s skill share increases. This implies that the secondary city’s skill share decreases. ■

Proof of Proposition 4. Because the equation

\[ \frac{L_1}{H_1} = \frac{\phi_1}{\phi_h} \left( \frac{w_{l,1}}{w_{h,1}} \frac{w_{l,2}}{w_{h,2}} \right)^\eta \]

must hold in equilibrium and \( \frac{w_{h,2}}{w_{l,2}} \) is constant, Proposition 3 implies Proposition 4. ■
Proof of Proposition 5. By the implicit function theorem,

\[ \frac{dw_{l,1}}{d\zeta_o} = -\frac{\partial L^D}{\partial \pi_1} - \frac{\partial L^S}{\partial \pi_1} \]

The denominator is negative, because

\[ \frac{\partial L^S}{\partial w_{l,1}} = \eta \frac{L_1}{H_1} \left( \frac{1}{w_{l,1}} + \frac{1}{L_1} \right) > 0 \]

and

\[ \frac{\partial L^D}{\partial w_l} = \frac{L_1}{H_1} \left( \frac{1}{\lambda_{w,1}} \frac{\partial \lambda_{w,1}}{\partial w_{l,1}} + \frac{1}{\lambda_{w,1}} \frac{\partial \lambda_{w,1}}{\partial w_{l,1}} - \sigma \left( \frac{L_1}{H_1} \frac{1}{w_{l,1}} + \frac{L_1}{H_1} \right) \right) < 0 \]

as \( \frac{\partial \lambda_{w,1}}{\partial w_{l,1}} < 0, \frac{\partial \lambda_{w,1}}{\partial w_{l,1}} < 0 \), and \( \hat{\lambda}_{c,1} \hat{\lambda}_{o,1} \left( \frac{\pi_1}{w_{l,1}} \right)^{\mu-1} < 1 \). For the numerator, it is negative when \( \eta \) is sufficiently big, even though \( \frac{\partial L^D}{\partial \pi_o} > 0 \) and \( \frac{\partial L^S}{\partial \pi_o} > 0 \). This is because \( \frac{\partial L^S}{\partial \pi_o} > 0, \lim_{\eta \to 0} \frac{\partial L^S}{\partial \pi_o} = 0 \), and \( \lim_{\eta \to \infty} \frac{\partial L^S}{\partial \pi_o} = \infty \). Thus, \( \frac{dw_{l,1}}{d\zeta_o} < 0 \) if \( \eta \) is sufficiently big. To see \( \frac{dw_{l,1}}{d\zeta_o} < 0 \), note that

\[ \frac{dw_{l,1}}{d\zeta_o} = -\frac{1}{\frac{\mu}{w_{l,1}} + \frac{\sigma}{w_{l,1}} \frac{\hat{\lambda}_{c,1} \hat{\lambda}_{o,1} \left( \frac{\pi_1}{w_{l,1}} \right)^{\mu-1}}{w_{l,1}} \left( \frac{1}{w_{l,1}} \right)^{\sigma} \left( \frac{w_{l,1}}{\pi_1} \right)^{\sigma-1}} \frac{1}{\frac{\partial \pi_1}{\partial \pi_1}} \]

Plugging this equation into (21), we have

\[ \frac{dw_{h,1}}{d\zeta_o} = -M_1 \frac{H_1}{H_1} \left( \frac{\mu + \eta \frac{\partial \pi_1}{\partial w_{l,1}}}{w_{l,1}} \frac{\partial \pi_1}{\partial \zeta_o} - \frac{1}{\lambda_{c,1}} \frac{\partial \lambda_{c,1}}{\partial \zeta_o} - \frac{1}{\lambda_{o,1}} \frac{\partial \lambda_{o,1}}{\partial \zeta_o} + \frac{L_1}{M_1} \frac{\partial \lambda_{o,1}}{\partial \zeta_o} \right) < 0 \]

because \( \frac{\partial \pi_1}{\partial \zeta_o} > 0, \frac{\partial \lambda_{c,1}}{\partial \zeta_o} < 0, \frac{\partial \lambda_{o,1}}{\partial \zeta_o} < 0, \frac{\partial \lambda_{o,1}}{\partial \zeta_o} > 0 \), and the denominator in the bracket equals \( \frac{\partial L^S}{\partial \pi_1} - \frac{\partial L^S}{\partial \pi_1} > 0 \). □

Proof of Proposition 6. In equilibrium, land supply must equal land demand. Thus,

\[ \frac{Q^P}{Q^2} = \frac{Q^S}{Q^2} \]

46
Let $r = \frac{\varrho_1}{\varrho_2}$ be the relative rent. By the implicit function theorem, 

$$\frac{dr}{d\zeta_o} = -\frac{\partial Q^D_1 / \partial Q^D_2}{\partial r} - \frac{\partial Q^S_1 / \partial Q^S_2}{\partial r}$$

The denominator is negative, because 

$$\frac{\partial Q^D_1 / \partial Q^D_2}{\partial r} = -\frac{Q_1}{Q_2} \left( \frac{1 + \eta}{r} \frac{w_{h,2} \phi_h + w_{l,2} \phi_l}{w_{h,2} (\phi_h - H_1) + w_{l,2} (\phi_l - L_1)} \right) < 0$$

$$\frac{\partial Q^S_1 / \partial Q^S_2}{\partial r} = \chi r^{x-1} > 0$$

As for the numerator, note that $\frac{\partial w_{h,1}}{\partial \pi_1} \frac{\partial \pi_1}{\partial w_{h,1}} = -\frac{L_1}{H_1}$ by (12) and (16) and use (21). Then, the numerator can be written as 

$$\frac{Q_1}{Q_2} \left( \frac{1 + \eta}{w_{h,1} H_1 + w_{l,1} L_1} \left( H_1 \frac{\partial w_{h,1}}{\partial \pi_1} \frac{\partial \pi_1}{\partial \zeta_o} + (1 - \frac{w_{h,1} w_{l,2}}{w_{h,2} w_{l,1}}) L_1 \frac{\partial w_{l,1}}{\partial \pi_1} \frac{\partial \pi_1}{\partial \zeta_o} \right) \right)$$

Note that $\frac{\partial w_{h,1}}{\partial \pi_1} < 0$ and $\frac{\partial \pi_1}{\partial \zeta_o} > 0$. Also, $\frac{w_{h,1}}{w_{l,1}} > \frac{w_{h,2}}{w_{l,2}}$, because the primary city is more productive and computers and outsourcing tend to substitute for local unskilled workers. In addition, $\frac{\partial w_{h,1}}{\partial \pi_1}$ is decreasing in $\eta$ and is negative when $\eta$ is sufficiently big. Therefore, the numerator is negative as long as $\eta$ is sufficiently big. Then, $\frac{dr}{d\zeta_o} < 0$. ■

**Proof of Proposition 7.** For the primary city’s high-tech producer, each skilled worker is paired up with 

$$\left( \int_{i_{c,1}}^{\omega_1(1)} \int_{0}^{\omega_1^{-1}(i_c)} z_c(i_c; \zeta_c)^{\mu} di_c di_c + \int_{\omega_1(1)}^{1} \int_{0}^{1} z_c(i_c; \zeta_c)^{\mu} di_c di_c \right) \pi_1^{\mu} \left( \frac{1 - \psi}{\psi} \right)^{\sigma} \left( \frac{w_{h,1}}{\pi_1} \right)^{\sigma}$$

units of computers and $\hat{i}_{c,1} \hat{i}_{o,1} \left( \frac{1 - \psi}{\psi} \right)^{\sigma} \left( \frac{\pi_1}{w_{h,1}} \right)^{\mu} \left( \frac{w_{h,1}}{\pi_1} \right)^{\sigma}$ units of unskilled workers. For the secondary city’s high-tech producer, each skilled worker is paired up with 

$$\int_{i_{c,2}}^{1} z_c(i_c; \zeta_c)^{\mu} di_c \pi_2^{\mu} \hat{i}_{c,2} \left( \frac{w_{h,2}}{\pi_2} \right)^{\sigma}$$
units of computers and
\[
\hat{i}_{c,2} \left( \frac{1 - \psi}{\psi} \right)^{\sigma} \left( \frac{\pi_2}{w_{l,2}} \right)^{\mu} \left( \frac{w_{h,2}}{\pi_2} \right)^{\sigma}
\]
units of unskilled workers. In addition, note that \( \omega_1 (1) = q_c^{-1} \left( \frac{w_{h,2}}{\zeta_c} \right) = \hat{i}_{c,2} \). Thus, the number of computers per worker for the primary city’s high-tech producer can be written as
\[
\frac{\int_{\omega_1(1)}^{\omega_1^{-1}(i_c)} z_c (i_c; \zeta_c)^{-\mu} di_c}{\pi_1^{\sigma - \mu} w_{h,1}^{-\sigma} \left( \frac{1 - \psi}{\psi} \right)^{-\sigma} + \hat{i}_{c,1} \hat{i}_{o,1}^{-\mu} w_{l,1}^{-\mu}}
\]
and the number of computers per worker for the secondary city's high-tech producer can be written as
\[
\frac{\int_{i_c,2}^{1} z_c (i_c; \zeta_c)^{-\mu} di_c}{\pi_2^{\sigma - \mu} w_{h,2}^{-\sigma} \left( \frac{1 - \psi}{\psi} \right)^{-\sigma} + \hat{i}_{c,2} w_{l,2}^{-\mu}}
\]
Since
\[
\int_{i_c,1}^{\omega_1(1)} \int_{0}^{\omega_1^{-1}(i_c)} z_c (i_c; \zeta_c)^{-\mu} di_c > 0
\]
\[
\frac{\pi_1^{\sigma - \mu} w_{h,1}^{-\sigma}}{\pi_2^{\sigma - \mu} w_{h,2}^{-\sigma} < \hat{i}_{c,1} \hat{i}_{o,1}^{-\mu} w_{l,1}^{-\mu} < \hat{i}_{c,2} w_{l,2}^{-\mu}
\]
(22) is greater than (23). Moreover, the secondary city has unskilled workers who perform outsourced tasks or work for the low-tech producer. Thus, the number of computers per worker in the primary city is greater than that in the secondary city.

Proof of Corollary 1. Clearly,
\[
\frac{\partial}{\partial \zeta_c} \left( \frac{\pi_1}{\pi_2} \right) = \frac{\pi_1}{\pi_2} \left( \pi_1^{\mu - 1} \int_{i_c,1}^{\omega_1(1)} \omega_1^{-1} (i_c) z_c (i_c; \zeta_c)^{-\mu} q_c (i_c) di_c + \left( \pi_1^{\mu - 1} - \pi_2^{\mu - 1} \right) \int_{i_c,2}^{1} z_c (i_c; \zeta_c)^{-\mu} q_c (i_c) di_c \right) > 0
\]

Proof of Proposition 8. Given Assumption 2, \( \frac{\partial}{\partial \zeta_c} > 0 \) holding fixed \( w_{l,1} \), and the proofs for Points 1, 2, and 3 are analogous to the proofs for Propositions 3, 4, and 5. To show that
Point 4 is true, the implicit function theorem is again applied as in the proof for Proposition 6. Since the denominator is negative, \( \frac{dr}{dc} \) is negative as long as the numerator is negative. The numerator can be written as

\[
\frac{Q_1}{Q_2} \left( \frac{H_1 \partial \pi_1}{\partial \pi_2} \frac{\partial \pi_1}{\partial c} - \left( 1 - \frac{w_{1,1} w_{2,1}}{w_{1,2} w_{2,1}} \right) L_1 \frac{\partial \pi_1}{\partial c} \right) + \frac{(1+\eta)H_1}{w_{1,1}H_1+w_{1,2}L_1} \frac{\partial \pi_1}{\partial \pi_1} \frac{\partial \pi_1}{\partial c} - \frac{1}{w_{1,2}(\phi_c-H_1)+w_{2,2}(\phi_c-L_1)} \left( \phi_c - H_1 \right) \frac{\partial \pi_2}{\partial \pi_2} \frac{\partial \pi_2}{\partial c} \right)
\]

The expression is negative as long as \( \eta \) is sufficiently large. ■

References


