Volatility, Mortgage Default, and CMBS Subordination

Christopher Downing, Richard Stanton, and Nancy Wallace*

February 19, 2008

Abstract

This paper calculates loan-by-loan estimates of commercial real estate implied volatility using all commercial mortgages in 206 public CMBS deals from 1996 through 2005 — a total of over 14,000 loans. The implied volatilities average about 20–24% per annum, with some differences across property types. Using these implied volatilities, we compute the distribution of default rates for representative CMBS pools under realistic assumptions, and find that the subordination levels for recent vintages of CMBS imply a high likelihood of default for what are supposed to be investment-grade tranches.

*Please address correspondence to (Downing): Barclays Global Investors, 45 Fremont St., San Francisco, CA 94105. Phone: (415) 817-6108. Fax: 415-618-5666. E-mail: chris.downing@barclaysglobal.com. (Stanton): Haas School of Business, University of California at Berkeley, Berkeley, CA 94720. Phone: (510) 642-7382. E-Mail: stanton@haas.berkeley.edu. (Wallace): Haas School of Business, University of California at Berkeley, Berkeley, CA 94720. Phone: (510) 642-4732. Fax: (510) 643-7357. E-Mail: wallace@haas.berkeley.edu

1 Introduction

The recent crisis in the subprime residential mortgage market has caused many investors to question both the safety of mortgage-related assets and the reliability of the ratings awarded by the rating agencies. Tomlinson and Evans (2007), in a Bloomberg report on the subprime crisis, quote Satyajit Das, a former banker at Citigroup:

> "The models are fine. But they have an input problem. It becomes a number we pluck out of the air. They could be wrong, and the ratings could be misleading."

The same report quotes Brian McManus, head of CDO Research at Wachovia:

> "With CDOs, they underestimated the volatility of the subprime asset class in determining how much leverage was OK."

In this paper, we raise concerns about the possibility of a similar crisis in the the commercial mortgage-backed security (CMBS) market.[1] As shown in Figure 1, subordination levels fell significantly between 1996 and 2005 for all classes of CMBS bonds.[2] A common justification for this decline is that, in the early days of CMBS issuance, the rating agencies based their subordination models on very conservative default and loss assumptions because they lacked historical delinquency and foreclosure data, and that, subsequently, they have revised their views in light of recent loss experience.[3] However, while cumulative defaults were less than 4.4% of all outstanding CMBS principal from 1993 through 2004,[4] these default levels appear to represent a thirty year low, perhaps due to the unusually attractive interest rate environment of the period, or to sample selection bias in CMBS pooling. Indeed, Figure 2 reports results from Esaki (2002), showing that cumulative 10-year default levels in the 18 years prior to 1990 averaged almost 20%. Given this, combined with both the magnitude of the recent declines in subordination levels, and the severe recent underestimation of default rates in the subprime residential mortgage market, it is important to examine these claims, and the likely future default rates on CMBS, more critically.

The loans purchased for CMBS pools usually contain provisions that limit prepayment risk, so default is the primary source of risk for CMBS investors. Default is primarily driven

[1] The first draft of this paper was written in April 2006, well before the start of the residential subprime crisis.

[2] The subordination level is the maximum amount of principal loss on the underlying mortgage that can occur without a given security suffering any loss. For AAA rated tranches, subordination levels fell from 35.59% to 14.08%, and for BBB tranches, they fell from 14.34% to 3.72%.

[3] See Wheeler (2001)

[4] Loss severities remained near 40% on average [see O'Rourke, Story, and Zaffuto (2005) and Johnson and MacNeill (2005)]

by large property price drops, whose likelihood in turn depends on the underlying assets' volatility. Although volatility plays a role in some commercial mortgage valuation models [notably, Titman and Torous (1989) and Kau, Keenan, Muller, and Epperson (1990)], there have been few attempts to estimate an empirically reasonable value.[5] Common estimation methods include using stock market or REIT return volatility as a proxy, or using the volatility of real estate price indices. However, these measures are either not calculated from the assets we are really interested in (REIT and stock market volatility), or they are calculated from smoothed estimates of returns on diversified portfolios of assets (price indices). In either case, the estimates are likely to underestimate the true volatility substantially.

Until recently, there was not as immediate a need for volatility estimation in the commercial real estate market because, unlike the active markets in equity and fixed income derivatives, there were few traded derivatives based on commercial real estate. In recent years, however, the CMBS market has expanded rapidly. It has seen an average annual growth rate of about 18% since 1997, and now stands second only to commercial banks as a source of credit to the commercial real estate sector.[6] There are also newer products, such as index-based CMBS derivative securities.

We take advantage of this increased securitization by using the Titman and Torous (1989) two-factor mortgage valuation model to estimate loan-by-loan implied volatilities for commercial real estate returns, from a sample of 14,041 non-seasoned fixed-rate mortgages, securitized into 206 public CMBS deals from 1996 to 2005. Our estimation results imply volatilities for commercial real estate returns of 20% or above, substantially higher than the values that have previously appeared in the literature.[7]

We next examine the adequacy of CMBS subordination levels between 1997 and 2006. We use our volatility estimates, plus conservative assumptions about the correlation between different property price movements, as inputs to a CMBS valuation model similar to that used by McConnell and Singh (1994) for collateralized mortgage obligations (CMOs), and Childs, Ott, and Riddiough (1996) for CMBS. Using the model to compute expected default

[5]Although there is a related literature that considers the incidence and severity of commercial mortgage defaults (e.g., An and Deng (2007), Jarrow and Yildirim (2006), Seslen and Wheaton (2005), Ambrose and Sanders (2003), Ciochetti, Deng, Lee, Shilling, and Yao (2003), Archer, Elmer, Harrison, and Ling (2002), Vandell, Barnes, Hartzell, Kraft, and Wendt (1993), and Snyderman (1991)), these papers do not explicitly estimate the effects of volatility.

[6]By the end of the third quarter of 2007, outstanding CMBS funded $637.2 billion, commercial banks $1,186.2 billion, and insurance companies $246.2 billion of the total $2.41 trillion of outstanding commercial mortgages [see Federal Reserve Z.1 Release (Flow of Funds), Third Quarter 2007].

[7]The few existing studies of implied volatility predate the development of the modern CMBS market. Titman and Torous (1989) apply a two factor model using quoted mortgage contract rates (as opposed to transaction rates) from 1985 through 1987. Ciochetti and Vandell (1999) and Holland, Ott, and Riddiough (2000) both calculate implied volatilities from one-factor mortgage valuation models, using mortgage origination data from the mid 1970s to the early 1990s.

rates for representative CMBS pools, we find that current subordination levels imply default likelihoods for supposedly investment-grade tranches that are substantially in excess of those claimed by the rating agencies. This misclassification, which has become worse over the last few years, suggests that we may soon see defaults and losses on commercial mortgages and CMBS to rival those currently being experienced in the residential mortgage sector.

The remainder of this paper is organized as follows. Section 2 discusses the problems with many estimates of historical volatility for commercial real estate, and presents a structural modeling framework to estimate implied volatility. Section 3 discusses the characteristics of our loan-level data, and estimates loan-by-loan implied volatilities for commercial real estate. Section 4 compares the loss rates implied by our model to observed subordination levels over the last three years of CMBS origination. Section 5 concludes the paper.

2 Volatility Estimation

Volatility is critical in many economic settings, including asset pricing, portfolio allocation, and the choice between investment and consumption. As a result, there is a large literature studying both historical and implied volatilities across many different asset classes, including listed equities,[8] interest rates,[9] and exchange rates.[10] However, there have been few attempts to estimate the volatility of commercial real estate. There are several reasons for this, despite the importance of the market.[11] In particular, the assets do not trade continuously, so many fewer transaction prices are available than for other asset classes; and the heterogeneity of real estate assets makes it difficult to aggregate the data that are available.

There are three main methods that have been used to estimate real estate volatility, two based on historical data, and one using implied volatility, the focus of this paper. We here discuss each of these methods in turn.

[8] See, for example, Black (1976), Schwert (1989, 1990), Jackwerth and Rubinstein (1996), Dumas, Fleming, and Whaley (1998), Britten-Jones and Neuberger (2000), and Jiang and Tian (2005).

[9] For surveys of this literature see Chapman and Pearson (2001) and Dai and Singleton (2003).

[10] See, for example, Jorion (1995), Zhou (1996), Andersen, Bollerslev, Diebold, and Labys (2001b), and Pong, Shackleton, Taylor, and Xu (2004).

[11] Commercial real estate in the U.S. had a total market value of $12.30 trillion at the end of the third quarter of 2007. This compares with $10.58 trillion for corporate bonds and $22.45 trillion for corporate equity [The Federal Reserve Flow of Funds, B100-B103 Balance Sheets, L.4 Credit Market Debt, Third Quarter 2007, http://www.federalreserve.gov/Releases/Z1/current/z1.pdf].

2.1 Historical Volatility

NCREIF indexes. One common method for estimating real estate volatility is by calculating the volatility of a total returns series such as that published by NCREIF.[12] The top panel of Table 1 reports the volatility of various NCREIF total returns series from 1995 to 2005. Estimates range from 5.75% for office properties to 8.93% for multifamily. These values are, however, likely to be underestimates of the true volatilities of the underlying properties. First, the index returns are subject to appraisal smoothing. Second, the index is calculated from a somewhat diversified portfolio of real estate, rather than an individual property.[13]

REIT returns. An alternative approach is to use the volatility of REIT returns.[14] An advantage over using NCREIF total returns series is that REIT returns are based on true trading prices, without the appraisal biases described above. Since many REITs make extensive use of leverage, it is important to adjust for this. The second panel of Table 1 reports unlevered REIT volatilities by property-types.[15] The mean volatility ranges from a high of 16.8% for industrial REITs to a low of 10.7% for multifamily REITS. As shown, there is dispersion in this measure across REITs within each property type (standard deviations of the estimates range from 3.1% to 11.3%) although the interquartile ranges are comparable. As expected, the REIT volatility estimates exceed those computed using the NCREIF total returns series, and for most property-types are nearly twice as large. Nevertheless, because REITs hold portfolios of assets, these REIT volatility estimates are still likely to be downwardly biased estimates of the true property-specific volatility, due to the effect of diversification.

[12] See, for example, Titman, Tompaidis, and Tsyplakov (2004). The National Council of Real Estate Investment Fiduciaries (NCREIF) National Property total returns series is the most widely used benchmark of property-level commercial real estate investment performance in the United States. The NCREIF total returns series is compiled from both the capital and the income performance of NCREIF monitored properties.

[13] Measurements of real estate returns must contend with numerous impediments such as infrequent trading and heterogeneity of the assets [Geltner and Goetzmann (1998) and Tu, Yu, and Sun (2004)], selection bias in transaction-based indices [Fisher, Gatzlaff, Geltner, and Haurin (2003), Fisher, Geltner, and Webb (1994), and Gatzlaff and Geltner (1998)], and seasonality and appraisal smoothing in appraisal-based indices [Clayton, Geltner, and Hamilton (2001)]. For a more detailed discussion of the NCREIF indices, see Geltner and Goetzmann (1998).

[14] See, for example, Titman and Torous (1989).

[15] We identify REIT property groupings using the National Association of Real Estate Investment Trust (NAREIT) website. We compute firm-specific annualized standard deviations of total returns using data from CRSP, and firm-specific unlevered volatility, using data from Compustat, to scale the firm's leveraged return volatility by the firm's average annual leverage ratio (long-term debt to total market capitalization).

2.2 Implied Volatility

Implied volatilities have several advantages over the historical measures described above. In particular, they are based on market prices and, because they are computed at the loan-level, we can control for specific underwriting characteristics of the loan contracts such as the loan-to-value (LTV) ratio, the coupon, and the amortization and maturity structure.

Calculating implied volatilities for commercial mortgage values is similar to using option prices to infer implied volatility for an equity security, and requires a mortgage pricing model. We use the two-factor model first proposed by Titman and Torous (1989), in which the value of a mortgage, M, is a function of interest rates, r, property prices, p, and time, t. Interest rates are governed by the Cox, Ingersoll, and Ross (1985) model,

$$dr_t = \kappa(\theta_r - r_t)\,dt + \phi_r\sqrt{r_t}\,dW_{r,t}, \tag{1}$$

where κ is the rate of reversion to the long-run mean, θ_r, and ϕ_r governs interest rate volatility. The price of interest rate risk is determined by the product ηr_t. We estimate the following parameters for the interest rate process, using the methodology of Pearson and Sun (1989) and daily data on constant maturity 3-month and 10-year Treasury rates for the period 1968–2006:

$$\begin{aligned} \kappa &= 0.13131, \\ \theta_r &= 0.05740, \\ \phi_r &= 0.06035, \\ \eta &= -0.07577. \end{aligned}$$

Property prices follow the geometric Brownian motion process,

$$dp_t = (\theta_{p,t} - q_p)p_t\,dt + \phi_p p_t\,dW_{p,t}, \tag{2}$$

where $\theta_{p,t}$ is the expected return on the property, q_p is the net income (on an unlevered basis), and ϕ_p is the volatility of the property return. We assume $\theta_{p,t} = r_t + \mu$, where r_t is the risk-free interest rate and μ is the risk-premium (assumed constant) on the property type in question, discussed below.

For pricing, we use the "risk-neutral" process,

$$dp_t = (r_t - q_p)p_t\,dt + \phi_p p_t\,dW_{p,t}, \tag{3}$$

in which $\theta_{p,t}$ is replaced with r_t.

Given the above processes for interest rates and property prices, the value of a commercial mortgage $M(p_t, r_t, t)$ with maturity date $T > t$, paying coupon C, must satisfy the partial differential equation:

$$\begin{aligned} \frac{1}{2}\phi_r^2 r M_{rr} + \frac{1}{2}\phi_p^2 p^2 M_{pp} + \rho\phi_r\phi_p p\sqrt{r}M_{rp} + \left(\kappa(\theta_r - r) - \eta r\right) M_r & \\ + \left((r - q_p)p_t\right) M_p + M_t - rM + C &= 0, \end{aligned} \tag{4}$$

where $E\left[dW_r dW_p\right] = \rho\, dt$, subject to boundary conditions described in detail in Titman and Torous (1989). For our initial baseline estimates, we assume that $\rho = 0$, and later consider the implications of relaxing this assumption through a series of robustness checks. We solve the model numerically, using a finite difference method to value the security and also to determine the critical default boundary.[16] Given this valuation model, the implied volatility for a given mortgage is then determined (also numerically) by finding the value of ϕ_p at which the model prices a newly issued mortgage at par.

3 Empirical Estimation

3.1 Loan characteristics

The data used in this analysis consist of 14,041 non-seasoned fixed rate commercial loans, originated between 1996 and the first quarter of 2005 and then securitized into CMBS pools.[17] The loan data were obtained from the public access websites for two CMBS trustees: Wells Fargo Trust Services and LaSalle. The two hundred and six CMBS pools in the sample represent about 63% of all non-private placement CMBS deals that included more than one loan.[18]

Contract types and rates. The loan-level frequencies by year of origination are reported in Table 2. As shown, the loans are fairly evenly spread across vintages and property-types, with lower loan counts in 1996 and 2005 when there were fewer deals. Table 3 provides summary statistics for the loans. Their mean amortization periods and maturity terms are quite similar across property types. The amortization range is between 310.39 and 341.16

[16]For details of the finite difference method used, see Gourlay and McKee (1977) and Downing, Stanton, and Wallace (2005).

[17]We focus on non-seasoned loans, excluding loans that exceed twelve months of seasoning, because we only observe each loan's loan-to-value ratio at the pool origination date. We also exclude floating rate loans, which appeared primarily in the 1997 and 1998 vintage loans. The seasoning exclusion eliminates about three thousand loans, and the floating rate exclusion another twenty seven hundred loans. These exclusions, plus missing data, leave 14,041 loans in our loan-level sample.

[18]Little information is available on single loan CMBS deals, such as Rockefeller Center, because these tend to be 144(a) private placements.

months and principal due dates range between 120.44 and 130.45 months. The average loan contract rates vary between a low of 6.92% for multifamily properties and a high of 7.23% for industrial properties. The interquartile ranges of the contract rates are comparable across property types, although the office and retail loan contact rate distributions have a slightly higher standard deviation than multifamily and industrial.

Debt service coverage ratios and loan-to-value ratios. The debt service coverage ratios and loan-to-value ratios move with the average coupon rates. For example, the relatively higher average contract rates and lower loan-to-value ratios for industrial properties are offset by lower debt service coverage ratios on these loans. Similarly, the low coupons on multifamily mortgages and higher loan-to-value ratios are offset on average by higher debt service coverage ratios. Retail loans have the lowest debt service coverage ratios and highest average loan-to-value ratios, and their average contract rates are also higher. The office contract rates are somewhat below those of the multifamily and retail loans and the loan-to-value ratios are somewhat higher, offset by higher average debt service coverage ratios. Of course, these summary statistics somewhat mask the exact underwriting criteria for each loan. It is clear, however, that setting any two of these underwriting characteristics significantly constrains the magnitude of the third.[19] For this reason, we only focus on coupon levels and loan-to-value ratios in our empirical simulations.

Loan balances and fees. Table 3 also shows that there is considerable heterogeneity in the initial loan balances across the four property types. Office and retail mortgages tend to be larger and more variable than multifamily and industrial mortgages. The overall average pool size from 1996 through 2005 was about $1.2 billion dollars, so the single largest retail and office loans do not appear to represent a significant concentration of risk within their respective pools. The table also reports summary statistics for the fees associated with the loans in the sample, computed as the difference between the reported full coupon and the net coupon on each mortgage. The mean fee levels are about 8 basis points across all property types.

3.2 Parameters of the property price process

The key parameters for the property price process in equation (2) are $\theta_{p,t}$, the expected return on the property (equal to r_t, the risk-free interest rate, plus μ, the risk-premium) and q_p, the net income. As discussed above, we solve for the implied volatilities using the risk

[19]Multivariate regressions of the debt-service-coverage ratio by property type on loan contract terms explain between 74% and 82% of the variance in the debt-service coverage ratios.

neutral property price process, but we also need the risk premium, μ, in order to estimate default probabilities. Both q_p and μ are estimated from market data.

We compute the net income component, q_p, from the realized income returns, obtained from NCREIF, between the first quarter of 1978 and the first quarter of 2005.[20] These are reported in the upper panel of Table 4. The income component of commercial property returns is quite stable over time, resulting in estimated standard errors for all four property types between .07% and .08%. Based on these results, we set the net income component to:

$$q_p = \begin{cases} 7.90\% & \text{for office properties;} \\ 7.84\% & \text{for multifamily properties;} \\ 7.85\% & \text{for retail properties;} \\ 8.47\% & \text{for industrial properties} \\ 7.99\% & \text{for other properties.} \end{cases}$$

We estimate the risk premium, μ, using the average excess return for NCREIF properties over 90 day T-Bills, quarterly from 1978 to 2005. Results are shown in the lower panel of Table 4. It is well-known that direct estimation of expected returns or risk-premia from historical returns can require very long time-series [see Merton (1980)]. Nevertheless, other than office, the interquartile ranges are about 600 basis points and the estimated standard errors are all below 1% (ranging from 0.47% to 0.74%), giving us a reasonable degree of comfort with our estimates.[21] The estimates values of μ for each property type are:[22]

$$\mu = \begin{cases} 3.11\% & \text{for office properties;} \\ 5.79\% & \text{for multifamily properties;} \\ 4.22\% & \text{for retail properties;} \\ 4.26\% & \text{for industrial properties;} \\ 3.85\% & \text{for other properties.} \end{cases}$$

[20]The returns are measured as (*Net Operating Income*) $\div$ (*Beginning market value* + .5 $\times$ *capital improvements* $-$ .5 $\times$ *partial sales* $-$ .333 $\times$ *Net Operating Income*). The adjustments are made to: 1) account for the assumption that net operating income (NOI) is received at the end of each month during the quarter; 2) the assumption that capital improvements occur at mid-quarter; 3) the assumption that partial sales occur at mid-quarter; 4) the assumption that the NOI is received monthly so that the cash flow received from the NOI in effect reduces the average investment in the property by .333 of NOI. These measures are the average property-type specific income returns for the properties held in the investment portfolios of pension funds.

[21]Any error we make in estimating μ will affect our estimates of default likelihood, but not our estimates of implied volatilities.

[22]We use the excess return estimate for the national property series as our estimate for an "other" category of properties which includes hotels, healthcare, self-storage, among others that exist in CMBS pools. The NCREIF return series do not include these other categories.

3.3 Implied volatility estimates

Table 5 reports loan-by-loan implied volatility estimates, defined as the volatility which sets the price of a newly issued mortgage equal to par. Comparing these results with Table 1, the implied volatilities are well above both those computed from historical property total returns series and those computed by unlevering REIT returns. Similar to the unlevered REIT return volatility estimates, office and industrial properties exhibit the highest implied volatilities, at 26.9% and 27.2%, respectively. For retail properties, the average implied volatility is 23.7%, and for multifamily properties it is 22.7%. Although the standard deviations are quite similar, the interquartile range is about nine percentage points for office and industrial properties and about seventeen percentage points for multifamily properties.

Liquidity adjustment. One possible criticism of the results in Table 5 is that they do not control for the illiquidity of unsecuritized commercial mortgages, which would tend to lower their value relative to the solution of the (non-liquidity adjusted) Equation (4). To address this, we calculate each pool's liquidity premium as the spread between the weighted average coupon of the mortgages in the pool, net of servicing fees, and the coupon on the more liquid BBB-rated CMBS tranche of the pool. There are two reasons that this spread is a plausible measure of the illiquidity premium on unsecuritized commercial real estate loans. First, whole commercial mortgages and the BBB tranche of CMBS have the same weighted average lives.[23] Second, commercial real estate mortgages that are held in bank balance sheets as whole loans get 100% risk-based capital weights under Basel 1A proposals for U.S. Commercial Banks, corresponding to a BBB bond rating.[24] Panel A of Table 6 shows that the average illiquidity premium for the mortgages in our sample ranges between 46 basis points for office and retail mortgages and 47 to 48 basis points for industrial and multifamily mortgages, with an interquartile range of about 53 basis points for all property types.

Panel B of Table 6 reports a second set of implied volatility estimates, calculated after first adjusting for the illiquidity of the whole mortgage by subtracting its pool-specific liquidity adjustment from its coupon rate. This results in a reduction in the estimated implied volatilities of approximately three percent.

Comparison with prior results. With or without the liquidity adjustment, our implied volatilities are substantially larger than volatilities obtained from either NCREIF indexes

[23] Over the sample period, the average weighted average life at origination of BBB CMBS tranches is about ten years, versus 9.7 years for the commercial mortgages in our sample [Source: CMBS origination data from CMAlert for all U.S. CMBS originations from 1997 to 2005].

[24] See http://www.federalreserve.gov/generalinfo/Basel2/NPR_20060906/NPR/section_1.htm

or REIT return series. They are also higher than prior implied volatility estimates. For example, Titman and Torous (1989) estimate an implied volatility of 15.5%, and the average implied volatility estimates from the one-factor mortgage valuation models of Ciochetti and Vandell (1999) and Holland et al. (2000) were 17.9% and about 19% respectively (none of these measures was liquidity adjusted). Closer to our results, in a non-mortgage framework, Quigg (1993) applied a real option valuation model to back out implied commercial real estate volatilities in Seattle, Washington. Her implied asset volatility estimates, are similar to ours, ranging between 18% and 28%. Our volatility estimates substantially exceed Treasury volatility[25] and stock market index volatility, and are closer to the volatility of individual equity securities.[26]

3.4 Robustness

Equation (2) assumes that volatility is constant over time. We here examine the reasonableness of this assumption. First, Figure 3 displays the average implied volatility by property type by quarter. The estimated median implied volatilities and interquartile ranges for the four property types appear to be stable over the period, and, apart from a spike in volatility for multifamily properties in 2000 resulting from the homeownership boom generated by overheated subprime lending, there are no particular trends visible. Augmented Dickey-Fuller tests show no evidence of either nonstationarity or a time trend in the results.[27]

Implied volatility vs. contract characteristics. If our model is correct, and we have observed all relevant features of each mortgage, the variables used as inputs to the model should explain 100% of the variation in implied volatilities, and these, in turn, should not vary systematically with any variables not in the model. To check this, Table 7 reports the results of cross-sectional regressions of the estimated implied volatility for each mortgage on both the contractual characteristics included in our structural model and two additional important loan characteristics, the debt-service coverage ratio (DSCR) and loan size. The column labeled "Pooled" shows the regression results for the pooled sample; this specification includes property-type dummies. The next four columns of the table display regression results for property-specific regressions. The adjusted-R^2 statistics range from 0.95 to 0.96,

[25] Brealey, Myers, and Allen (2006) report annual standard deviations (using monthly returns from 1900–2003) of 2.8% for Treasury Bills, and 8.2% for longer maturity Government bonds.

[26] Brealey et al. (2006) report a volatility of 20.1% for a diversified portfolio of common stocks between 1900–2003. For individual stocks, Andersen, Bollerslev, Diebold, and Ebens (2001a) estimate annualized standard deviations of around 28% for the average stock, with variation in their sample between 22% and 42%.

[27] Results are available on request from the authors.

suggesting that these variables capture almost all of the variation in the implied volatilities. The coefficient on DSCR is statistically insignificant, and the coefficient on loan size, while statistically significant, is economically insignificant.[28] From these results, we conclude that the DSCR and, to a lesser extent, the loan size, are redundant once the loan-to-value ratio, the coupon, the amortization period and maturity have been taken into account.

Implied volatility vs. macroeconomic variables. As a second robustness check, we analyze possible macro-determinants of the time series characteristics of the implied volatility series by regressing quarterly changes in the liquidity adjusted volatilities on the contemporaneous quarterly changes in four macroeconomic variables: the ten year Treasury Rate; the slope of the Treasury curve, measured by the difference between the ten-year and one-year constant maturity Treasury rates;[29] the current dollar U.S. GDP;[30] and the American Council of Life Insurers measure of the fraction of life insurance company portfolios that are 60 days in arrears.[31] Results are reported in Table 8. The uniformly small F statistics indicate that the null on the joint hypothesis test of zero valued coefficients should be accepted. Although it appears that there is a statistically significant and positive effect of the change in the ACLI delinquency rate on the average industrial implied volatility, overall these regressions indicate that the macro channels do not drive changes in volatility. These results, again, are supportive of the representation of property price dynamics assumed in Equation (2).

Correlation between property prices and interest rates. As a third robustness check, we analyze the impact of our assumption that property returns and interest rates are uncorrelated ($\rho = 0$). Table 9 displays estimates of liquidity-adjusted implied volatilities where we relax this zero correlation assumption. The first column displays the average implied volatilities by property type where we set $\rho = -0.75$, and the third column displays the results for $\rho = 0.75$. While the implied volatilities are somewhat inversely related to the correlation coefficient, what little evidence there is in the literature on the sign and magnitude of the correlation between interest rates and property returns indicates an indeterminate sign but a low absolute value of the correlation parameter [Titman and Torous (1989)]. Hence, varying ρ within a plausible range leads to only small changes in our implied volatility estimates.

[28] The coefficient of 0.0018 for the pooled sample implies that a change in the loan size from \$5 million–approximately the mean loan size in the sample—to \$50 million leads to a change in the implied volatility of approximately 0.4%.

[29] http://www.federalreserve.gov/releases/h15/data.htm

[30] http://www.bea.gov/national/index.htm#gdp

[31] These data were obtained from the American Council of Life Insurers

In summary, our robustness checks broadly support the assumptions of our structural model of commercial mortgage valuation. There is no significant evidence in the implied volatility estimates of nonstationarity, a time-trend, or any significant relation to variables outside the model.

4 CMBS Subordination and Default Rates

Given the dynamics of property prices in Equation (2), and our estimates for property-specific implied volatilities, we now turn to estimating the distribution of defaults for single mortgages and pools of mortgages, and evaluating the adequacy of current CMBS subordination levels.[32] Pool-level losses are related both to expected defaults on the individual loans and to the dispersion of the total pool-level default around this level. Higher expected mortgage default rates, driven by higher property-level volatility on the underlying real estate, mean more required subordination for all tranches. Dispersion around the expected default level, determined by the correlation between the individual mortgages, also affects the required subordination levels. The correlation between mortgages does not affect the total value of all CMBS tranches,[33] However, it does affect the values of individual tranches. In general, more dispersion (more correlation) lowers the value of safer tranches, and increases the value of extremely risky tranches.[34] The tranches most adversely affected by greater dispersion of mortgage default would not be the AAA securities, which are protected even if defaults are substantially higher than expected, but the securities slightly lower down in priority, such as BBB.

4.1 Calibration

Correlation between returns. In order to model correlation between the properties in a mortgage pool, we split the return shocks for each property into two components, a common component shared across all properties, and a property-specific component, whose volatility

[32] While several prior authors have looked at either default or CMBS valuation, we are aware of no prior research that addresses the sufficiency of observed subordination levels.

[33] Ignoring spreads and/or liquidity differences, the total CMBS cash flow equals the total mortgage cash flow, and the value of each mortgage does not depend on correlation.

[34] The dependence on dispersion/correlation arises because tranching makes CMBS payoffs nonlinear in the default rates of the underlying mortgages. Hence, by Jensen's Inequality, the expected cash flow to a CMBS is not equal to its cash flow at the expected default rate on the underlying mortgages, the difference depending on the volatility of the cash flows. As an example, suppose that a CMBS structure protected against losses up to 10%, and the expected loss on the mortgages was 10%. If the default rate were certain, then the CMBS would experience a 0% default rate. If the default rate were uncertain, and, say, had a fifty percent chance of a 0% or 20% default realization, the CMBS would have an expected loss of 5% of underlying principal.

varies by property type. More precisely, we simulate draws from the following system:

$$
\begin{aligned}
dr_t &= \kappa(\theta_r - r_t)\, dt + \phi_r \sqrt{r_t}\, dW_{r,t}, && (5)\\
dp^j_{i,t} &= (\theta^j_{p,t} - q^j_p) p^j_{i,t}\, dt + \phi p^j_{i,t}\, dW_t + \phi^j_p p^j_{i,t}\, dW^i_t, && (6)
\end{aligned}
$$

where $p^j_{i,t}$ is the price of property i, of type j (where $i = 1, 2, 3, 4, 5$ indexes apartment, office, retail, industrial, and all other properties, respectively), dW_t is common across all properties, and dW^i_t is an independent shock for each property.

As discussed earlier, the standard deviation of the NCREIF total returns series is unsuitable as an estimate of property-level asset volatility, both because it is calculated from the returns on a diversified portfolio, and because the return measurements suffer from appraisal bias and smoothing. The former feature, however, makes the index useful for calibrating a common component to property return volatilities (though smoothing will impose a downward bias on this calibrated value). The systematic volatility, ϕ, is set equal to the volatility of the national total returns series, 7.019%. The idiosyncratic volatility terms, ϕ_{p_i}, are set to obtain the correct total volatilities given in Table 6. For example, the idiosyncratic volatility for office properties is set to ϕ_{p_i}=0.2274; with the given setting of ϕ, the total volatility for office properties is

$$\sqrt{0.07019^2 + 0.2274^2} = 0.238,$$

as shown in Table 6. The decomposition of the individual property volatility into a systematic and idiosyncratic component produces a modest amount of positive correlation across the property types, as shown in Table 10. In general, the correlation in returns across each pair of different property types is in the range of 0.085 to 0.116. The correlations are low because the proportion of systematic volatility to idiosyncratic volatility is relatively low, perhaps in part owing to the smoothed nature of the national property returns series. It is of course straightforward to modify the setting of ϕ to boost the correlations across the property types, for example in a stress-testing procedure, or to reflect a different view on the likely magnitude of the correlation parameter.

Contract terms and risk-free rate. Finally, it remains to define the mortgage contract parameters and initial conditions for the simulations. Our ultimate aim is to benchmark our implied default rates against the subordination levels observed on recently originated CMBS pools. With this aim in mind, we set the contract parameters and initial conditions equal to their sample averages for the mortgages in the latter part of our sample. Based on the data in Table 3, we set the term of the mortgages in our simulation to 10 years, with a 30 year amortization period. We set the coupon rates to their sample averages and the initial risk-free rate to 2.42%, the average bill rate for 2004 and 2005.

Distribution of property types. Table 11 presents a breakdown of the distribution of the four property types—office, multifamily, retail, and industrial—in our sample of 206 CMBS pools. As shown, there are pronounced trends in the property-type distributions over time. For example, multifamily mortgages represented a high of 27.16% of the collateral in CMBS pools in 1996 and fell to a low of 13.46% in 2005. The decline in multifamily loans reflects relatively higher default incidence in this sector and the recent aggressive growth of the government sponsored enterprises' (Fannie Mae and Freddie Mac) multifamily securitization programs. Mortgages on office properties represented from 13.25% of CMBS collateral in 1999 to a high of about 33.47% in 2005. Average allocations for industrial properties only exceeded 10% at the height of the dot-com boom.[35]

Simulation Details. To estimate the default behavior of pools of mortgages, we first create a simulated pool, containing 100 mortgages, with types chosen to match the average proportions shown in Table 11: 25 apartment, 20 office, 30 retail, 10 industrial, and 15 "other" (proxied by national averages). For each mortgage, we randomly draw an LTV so that we match the sample mean and standard deviation of the origination LTV ratios for the property's type (these LTVs are reported in Table 3). Within each property type, though the mortgages differ in their initial LTV ratio, they share the sample average coupon level, term, and amortization schedule.

Given the composition of the pool, we now make 5,000 draws from the system of equations (5) and (6), and keep track of the frequency with which the joint interest rate and property price process moves into the region where each borrower optimally chooses to default.[36] We compute the default frequency by quarter by computing the proportion of the original 100 mortgages in the pool that default, and then calculate cumulative default rates by summing the quarterly default rates. Note that the common shock to the property return processes induces correlation in defaults across the mortgages in the pool.

[35] In the first two years of the sample, hotels made up about 14% of the pools. However, their share steadily dropped to a low of 2.7% in 2002 due to high defaults in this sector. By 2003, hotels accounted for almost one-third of new CMBS defaults and, by year end 2004, hotel mortgages accounted for the largest sector of cumulative losses at $2.7 billion [O'Rourke et al. (2005)]. Because there are no hotel REITs or NCREIF series for hotels we do not model hotels separately.

[36] The default boundary for each loan is determined as part of the solution of Equation (4).

4.2 Results

Expected Default Rates. Figure 4 shows the distribution of cumulative default rates under our implied volatility measures. The solid line indicates the median cumulative default rate across the simulations, the dashed lines show the approximate location of the 25^{th} and 75^{th} percentiles, and the dotted lines show the 5^{th} and 95^{th} percentiles, respectively. As can be seen, for approximately the first two years from origination, there are virtually no defaults, consistent with the fact that, by and large, the simulated loans carry low LTV levels. Starting around year two, defaults begin to ramp up, with the median cumulative default rate reaching 4.7% 15 quarters after origination, with an interquartile range of 6.5% to 2.3%. At the end of the 10-year horizon, the median cumulative default rate is 21%, with an interquartile range from 15% to 29%. Applying a 40% severity-of-loss rule, the 21% median cumulative default rate reported in Figure 4 implies a median 8.4% loss rate over a ten year horizon.

Our estimated median 10 year cumulative default rate of 21%, while higher than recently observed default rates, is nevertheless similar to realized default rates over a longer horizon. This estimate is consistent with the findings of Esaki (2002), previously reported in Figure 2, who found an average cumulative default rate over a 10-year horizon of 18.4% for commercial real estate mortgages originated by insurance companies [see also Esaki (2003)]. As shown in the Figure, default rates between 1991 and 1995 were unusually low compared with prior periods.

CMBS subordination levels. Using this simulated distribution of defaults, Table 12 reports estimates for the percentage of CMBS loan defaults that would be required to generate losses for a range of tranches from very risky, class B, to very safe, class AAA. For each class we report the weighted average subordination levels observed in conduit and fusion CMBS pools originated in 2004, 2005, and 2006. These data were obtained from CMAlert, and they reflect the universe of conduit and fusion CMBS originated in the United States over this period. The average subordination levels reported in Table 12 are based on 61 pools in 2004, 63 pools in 2005, and 66 pools in 2006. We also assume that loan losses will stay at the historical average of 40% Johnson and MacNeill (2005). Given this loss rate and observed subordination levels for the BBB tranches, the percent of defaults that would be required to generate losses to the BBB tranche investors would be 13.3% in 2004, 11.8% in 2005, and 11.0% in 2006. As shown in Figure 4, all of these values are well below the median default rate generated by our model, 21%. Given our cumulative default estimates over a ten year horizon, defaults large enough to hit the BBB tranches for the 2004, 2005, and 2006 vintages of subordination would be expected to occur with probability 81%, 84% and 87%

respectively.

To determine how large subordination levels ought to be, note first that, ignoring dispersion in the realized default levels, even if we always saw the median default rate of 21%, combined with our assumed loss given default of 40%, we would need an 8.4% subordination level to avoid defaults on the BBB securities. However, the effect of dispersion magnifies this effect. Taking both the expected default and dispersion into account, we would want 17.2% subordination for the BBB in 2006. This subordination level corresponds to the subordination levels observed in the late 1990's.

5 Conclusions

This paper adds to our understanding of asset market volatility by systematically estimating the volatility of commercial real estate using loan-level data on the more than 14,000 commercial mortgages in 206 public CMBS deals from 1996 through 2005. We find that, on average, the implied volatility for the properties in our sample is 22%, with differences across property types. This is substantially higher than prior estimates in the literature, both those based on historical data and those based on implied volatilities.

Our estimates have immediate implications for CMBS markets, because the likelihood of CMBS default is closely related to the volatility of the underlying assets. While subordination levels on CMBS have systematically declined over the past few years, our implied volatility estimates have remained roughly constant. Based on current subordination levels, we find that many CMBS tranches, such as the BBB pieces, have much higher default likelihoods than those supposedly implied by their credit ratings. While the reduction in subordination levels is not inconsistent with the low levels of default we have seen in recent years, it is important to note that we have seen much higher default rates, roughly consistent with those predicted by our model, only a few years earlier.[37] It is thus likely that, over the next ten years, we will see significant defaults and losses on investment grade CMBS bonds from recent vintage CMBS pools. Indeed, even as we write, the Wall Street Journal reports that Lehman Brothers is expected to take a write-down "in the $1.3 billion range", on about $90 billion of debt securities, of which "nearly $39 billion are commercial real estate loans."[38]

[37] See Esaki (2002) and Esaki (2003).

[38] See "Now, Lehman Gets Pelted", Heard on the Street, WSJ, February 19, 2008.

Table 1: Total Return Volatility Estimates Using NCREIF Total Return Indexes and Unlevered Firm-level REITs Total Returns over the Period 1995 through 2005

The table presents the computed values for the volatility of total returns from two alternative data sources. The upper panel of the table reports the annualized total returns volatility computed from the NCREIF quarterly total returns indexes by property type from 1978 through 2006. The lower panel reports the annualized unlevered total returns volatility from 1996 through 2006 for samples of REITs for the reported property types. The data for the upper panel were obtained from NCREIF and from the lower panel from CRSP and Compustat.

	No. of Obs.	Mean (%)	Std. Dev. (%)	25^{th} Percentile (%)	75^{th} Percentile (%)
		NCREIF Index Volatilities			
Office Index		5.75			
Multifamily Index		8.93			
Retail Index		5.80			
Overall Index		5.79			
		Unlevered Firm-level REIT Volatilities			
Office REITs	20	15.5	6.6	10.7	15.9
Multifamily REITs	18	10.7	3.1	8.1	12.4
Regional Mall REITs	9	13.2	10.2	7.3	12.7
Retail REITs	19	15.5	9.4	11.7	15.6
Industrial REITs	5	16.8	11.3	10.0	14.8

Table 2: Loan-Level Frequencies by Year of Origination and Property Type

The table presents the loan-level frequencies for the non-seasoned industrial, multifamily, office, and retail commercial mortgages by the year of their origination. The sample is all non-seasoned mortgages with complete origination and performance data that were securitized in one of the 206 CMBS deals originated between 1996 and the beginning of 2005. The data were obtained from the websites of two trustees: Wells Fargo Trust Services and LaSalle.

Year of Origination	Office	Multifamily	Retail	Industrial	Total
1996	27	41	9	14	91
1997	589	596	222	123	1530
1998	812	946	372	227	2357
1999	212	284	115	134	745
2000	267	227	211	111	816
2001	693	703	453	281	2130
2002	788	752	444	214	2198
2003	766	665	364	186	1981
2004	807	574	458	92	1931
2005	120	49	66	27	262
Total	2714	4837	5081	1409	14041

Table 3: Loan-Level Summary Statistics by Property Type

The table presents summary statistics for the contract terms of the 14,041 commercial mortgages in our sample. These mortgages are all non-seasoned mortgages with complete origination and performance data that were securitized in one of the 206 CMBS deals originated between 1996 and the beginning of 2005. The data were obtained from the websites of two trustees: Wells Fargo Trust Services and LaSalle.

Property Type	Loan Characteristic	Mean	Std. Dev.	25^{th} Percentile	75^{th} Percentile
Office	Amortization Term (Months)	327.77	64.66	300.00	360.00
Multifamily		341.16	50.31	360.00	360.00
Retail		320.58	64.52	300.00	360.00
Industrial		310.39	65.70	300.00	360.00
Office	Maturity (Months)	120.44	33.70	120.00	120.00
Multifamily		123.29	39.60	120.00	120.00
Retail		130.45	39.60	120.00	120.00
Industrial		127.21	34.78	120.00	120.00
Office	Contract Rate (%)	6.95	1.08	5.98	7.68
Multifamily		6.92	1.02	6.09	7.55
Retail		6.98	1.08	6.01	7.72
Industrial		7.23	1.00	6.60	7.88
Office	Original LTV Ratio (%)	67.40	11.25	63.27	74.69
Multifamily		71.40	13.26	66.09	79.32
Retail		69.50	11.02	66.09	79.32
Industrial		66.30	11.40	61.09	74.20
Office	Debt Service Coverage Ratio	1.50	0.34	1.31	1.57
Multifamily		1.55	0.76	1.27	1.52
Retail		1.45	0.29	1.29	1.52
Industrial		1.49	0.34	1.30	1.57
Office	Initial Loan Balance ($)	11,146,586	19,453,026	2,800,000	115,000,000
Multifamily		5,704,454	6,361,522	1,880,000	7,000,000
Retail		7,673,014	13,223,980	2,300,000	8,000,000
Industrial		5,546,275	6,172,538	2,000,000	6,500,000
Office	Fees (%)	0.08	0.046	0.042	0.116
Multifamily		0.08	0.046	0.042	0.121
Retail		0.08	0.048	0.042	0.122
Industrial		0.08	0.050	0.042	0.123

Table 4: Summary Statistics for the National Council of Real Estate Investment Fiduciaries (NCREIF) Income Returns and Excess Return for Properties by Type and Year

The table presents summary statistics for NCREIF measures for income and excess total return for all properties held in portfolio from the first quarter of 1978 through the first quarter of 2005. The returns are reported as annualized quarterly returns over the period. These measures are the average property-type specific income returns for the properties held in the investment portfolios of pension funds. The data were obtained from NCREIF.

Property Type	Mean (%)	Std. Error (%)	25^{th} Percentile (%)	75^{th} Percentile (%)
q_p = Income Return				
Office	7.90	0.08	7.13	8.71
Multifamily	7.84	0.08	7.17	8.54
Retail	7.85	0.08	7.19	8.50
Industrial	8.47	0.07	7.90	9.01
National (Other Category)	7.99	0.07	7.36	8.67
μ = Excess Return = Income Return + Capital Return - 90-day Treasury Yield				
Office	3.11	0.75	-1.68	8.50
Multifamily	5.79	0.47	2.23	8.63
Retail	4.22	0.60	-0.45	6.90
Industrial	4.26	0.51	1.77	7.44
National(Other Category)	3.85	0.52	1.52	6.91

Table 5: Implied Volatilities by Property Type

The table presents the computed implied instantaneous volatilities for our sample of loans. The implied volatility is defined for each loan as the value of ϕ_p in Equation (4) that sets the initial value of the loan equal to par.

	No. of Obs.	Mean (%)	Std. Dev. (%)	25^{th} Percentile (%)	75^{th} Percentile (%)
Office	2,227	26.9	7.8	21.6	30.1
Multifamily	4,028	22.7	7.8	17.8	34.7
Retail	4,205	23.7	7.9	18.6	37.0
Industrial	1,234	27.2	7.7	22.4	31.6

Table 6: Liquidity Adjusted Implied Volatilities by Property Type

The table presents the computed implied instantaneous volatilities for our sample of loans. The implied volatility is defined for each loan as the value of ϕ_p in Equation (4) that sets the initial value of the loan equal to par. The estimates in this table are calculated after making an adjustment for the liquidity premium embedded in the mortgage coupon rate.

Panel A: Descriptive Statistics for the Illiquidity Measure

	No. of Obs.	Mean Basis points	Std. Dev. Basis points	25^{th} Percentile Basis points	75^{th} Percentile Basis points
Office	2,227	45.6	36.8	17.3	65.1
Multifamily	4,028	48.2	38.2	17.9	70.1
Retail	4,205	46.0	36.4	17.4	70.0
Industrial	1,234	47.4	37.2	16.8	70.4

Panel B: Liquidity-Adjusted Volatility

	No. of Obs.	Mean (%)	Std. Dev. (%)	25^{th} Percentile (%)	75^{th} Percentile (%)
Office	2,227	23.8	7.8	18.8	27.4
Multifamily	4,028	19.7	7.6	14.9	21.9
Retail	4,205	21.5	7.6	16.4	25.1
Industrial	1,234	24.1	7.8	18.8	28.2

Table 7: Regressions of Liquidity-Adjusted Loan-Specific Implied Volatilities on Input Parameters and Other Variables

The table presents diagnostic results for the determinants of our computed liquidity-adjusted loan-specific implied volatilities. We consider the relative importance of all contract terms that are included in the model, along with terms such as the debt service coverage ratio and loan size that are outside the model.

	Sample				
Variable	Pooled	Office	Retail Family	Multi-	Industrial
Intercept	0.5676***	0.5910***	0.5970***	0.5332***	0.5698***
	(0.0042)	(0.0100)	(0.0070)	(0.0066)	(0.0116)
Retail	0.0044				
	(0.0006)				
Office	0.0171***				
	(0.0006)				
Manufacturing	0.0126***				
	(0.0006)				
Loan-to-Value	-0.4605***	-0.4537***	-0.4759***	-0.4346***	-0.4817***
	(0.0021)	(0.0047)	(0.0036)	(0.0035)	(0.0059)
Coupon	0.0593***	0.0592***	0.0581***	0.0601***	0.0599***
	(0.0003)	(0.0008)	(0.0005)	(0.0004)	(0.0008)
10 Year CMT	-0.0492***	-0.0502***	-0.0483***	-0.0493***	-0.0481***
	(0.0003)	(0.0009)	(0.0006)	(0.0005)	(0.0010)
Mortgage Maturity	-0.0003***	-0.0004***	-0.0004***	-0.0003***	-0.0003***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Amortization Period	-0.0005***	-0.0006***	-0.0006***	-0.0005***	-0.0005***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Debt Coverage	-0.0008	0.0001	-0.0034	-0.0040	-0.0044
	(0.0007)	(0.0016)	(0.0023)	(0.0024)	(0.0020)
Log of Loan Size	0.0018***	0.0023***	0.0022***	0.0013**	0.0016**
	(0.0002)	(0.0004)	(0.0009)	(0.0006)	(0.0007)
Adj.-R^2	0.95	0.95	0.95	0.95	0.96
F Statistic	20,288.70	5,018.79	9,935.51	10,327.6	3,584.29
Number of Obs.	10,191	1,914	3,629	3,596	1053

Table 8: Regression of Estimated Implied Volatilities by Property Type on Macro-Fundamentals (Quarterly Data 1996-2005)

The table reports ordinary least squares estimation results for quarterly changes in the implied volatilities on contemporaneous quarterly changes in macro-fundamentals. Residual diagnostics for each regression are reported, along with an F test of the null hypothesis that the estimated coefficients are jointly zero.

Variables	Office	Multifamily	Retail	Industrial
Intercept	-0.020	-0.011	-0.017	0.006
	(0.014)	(0.017)	(0.015)	(0.015)
Change in Ten Year Treasury Rate	-0.015	-0.009	-0.017	0.001
	(0.020)	(0.024)	(0.021)	(0.020)
Change in Term Structure Slope (10Yr CMT - 1Yr CMT)	0.023	0.007	0.015	.010
	(0.018)	(0.022)	(0.019)	(0.020)
Log Change in GDP	3.632	0.829	2.428	-0.699
	(3.489)	(4.237)	(3.724)	(3.853)
Change in ACLI Delinquency Rate	4.582	-6.588	1.096	9.193**
	(4.042)	(4.910)	(4.316)	(4.465)
F Statistic	0.660	0.756	0.216	1.186
Breusch-Godfrey Serial Correlation LM Test	0.66	0.12	0.11	0.16
Akaike Criterion	-3.04	-3.02	-3.27	-3.21

Table 9: Implied Volatilities and the Correlation between Interest Rates and Property Returns

The table shows the mean implied liquidity-adjusted volatility estimates for models where we set the correlation between interest rates and property returns (ρ) equal to -0.75, 0, and 0.75, respectively. The figures shown in the table are the average implied volatility estimates over the indicated property types.

Property	ρ		
Type	-0.75	0.00	0.75
Office	27.4	23.8	20.8
Multifamily	23.2	19.8	16.3
Retail	24.3	21.5	17.5
Industrial	27.7	24.1	20.8

Table 10: Correlations of Simulated Property Returns

The table displays the correlations between property returns of different types. Following the notation of Equation 6, the correlations are computed as

$$\rho_{ij} = \frac{1}{\sqrt{(1+\frac{\phi_{pi}^2}{\phi_p^2})(1+\frac{\phi_{pj}^2}{\phi_p^2})}}.$$

	Multifamily	Industrial	Office	Retail	Other
Multifamily	0.127				
Industrial	0.104	0.085			
Office	0.105	0.086	0.087		
Retail	0.116	0.095	0.096	0.107	
Other	0.112	0.092	0.093	0.103	0.099

Figure 1: CMBS Weighted Average Subordination Levels

This Figure plots the annual average percentage of subordination by bond class for the 206 CMBS pools in our sample.

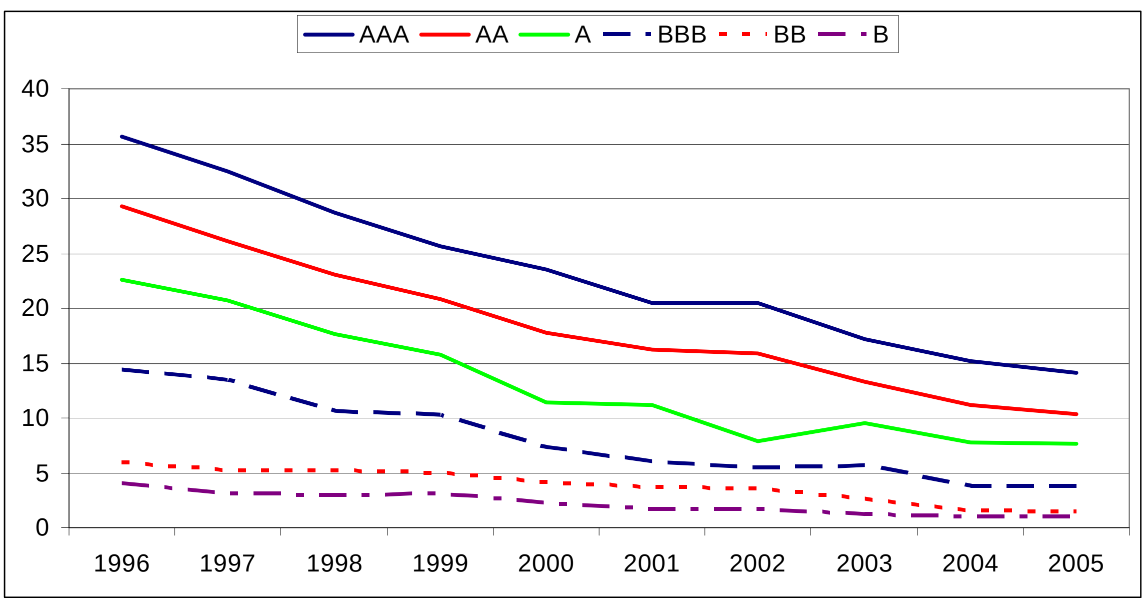


Table 11: Average Principal Allocations by Property-Type Composition for the 206 CMBS Pools

The table presents the property type composition for the sample of 206 CMBS deals that were originated between 1996 and the beginning of 2005. These deals are all public, or SEC registered CMBS pools with more than one loan. The data were obtained from the websites of two trustees: Wells Fargo Trust Services and LaSalle.

Orig. Year	Deals		Office % of Pool	Multifamily % of Pool	Retail % of Pool	Industrial % of Pool
1996	12	Mean	14.37	27.16	30.20	7.82
		Std. Dev.	10.41	10.93	12.64	6.68
1997	14	Mean	19.95	23.36	30.07	6.30
		Std. Dev.	9.40	11.63	8.28	3.89
1998	18	Mean	13.58	26.83	35.21	5.49
		Std. Dev.	5.54	9.78	24.35	2.59
1999	15	Mean	13.25	30.69	21.37	9.54
		Std. Dev.	9.43	9.91	10.94	10.08
2000	17	Mean	25.63	21.80	33.06	9.63
		Std. Dev.	10.03	7.05	21.16	6.97
2001	28	Mean	31.54	20.81	29.37	10.73
		Std. Dev.	15.42	9.89	7.80	7.56
2002	29	Mean	26.59	23.54	35.88	7.63
		Std. Dev.	15.99	9.14	6.69	5.92
2003	29	Mean	26.61	15.57	39.32	6.38
		Std. Dev.	7.47	9.07	9.32	6.64
2004	38	Mean	30.29	17.41	33.50	3.45
		Std. Dev.	10.81	7.78	9.87	2.39
2005	6	Mean	33.47	13.46	35.64	2.93
		Std. Dev.	10.87	6.86	6.27	3.28

Table 12: Implied Default Rates

The table displays estimates for the percentage of loan defaults in a pool that would be required to generate losses for tranched classes with ratings from AAA to B. The reported weighted-average subordination levels are those observed for the universe of CMBS pools originated in 2004, 2005 and 2006. The subordination structure for these pools were obtained from CMAlert.

Class	Percentage Subordination %	Historical Loss Severity %	Defaults Required for Loss %
	2004 CMBS Conduit Pools - Number of Pools = 61		
AAA	15.3	40.0	38.3
AA	11.8	40.0	29.5
A	8.8	40.0	22.0
BBB	5.3	40.0	13.3
BB	3.1	40.0	7.8
B	1.7	40.0	4.3
	2005 CMBS Conduit Pools - Number of Pools = 63		
AAA	23.2	40.0	58.0
AA	10.8	40.0	27.0
A	8.1	40.0	20.3
BBB	4.7	40.0	11.8
BB	2.7	40.0	6.8
B	1.7	40.0	4.3
	2006 CMBS Conduit Pools - Number of Pools = 66		
AAA	26.2	40.0	65.5
AA	10.2	40.0	25.5
A	7.7	40.0	19.3
BBB	4.4	40.0	11.0
BB	2.5	40.0	6.3
B	1.6	40.0	4.0

Figure 2: Historical Realized Default Levels

This Figure plots the lifetime default rates (loan counts) by origination cohort for 116,595 commercial real estate loans held by a sample of major insurance companies. These default rates were reported in "Commercial Mortgage Defaults: 1972-2000," by H. Esaki, *Journal of Real Estate Finance*, Winter, 2002.

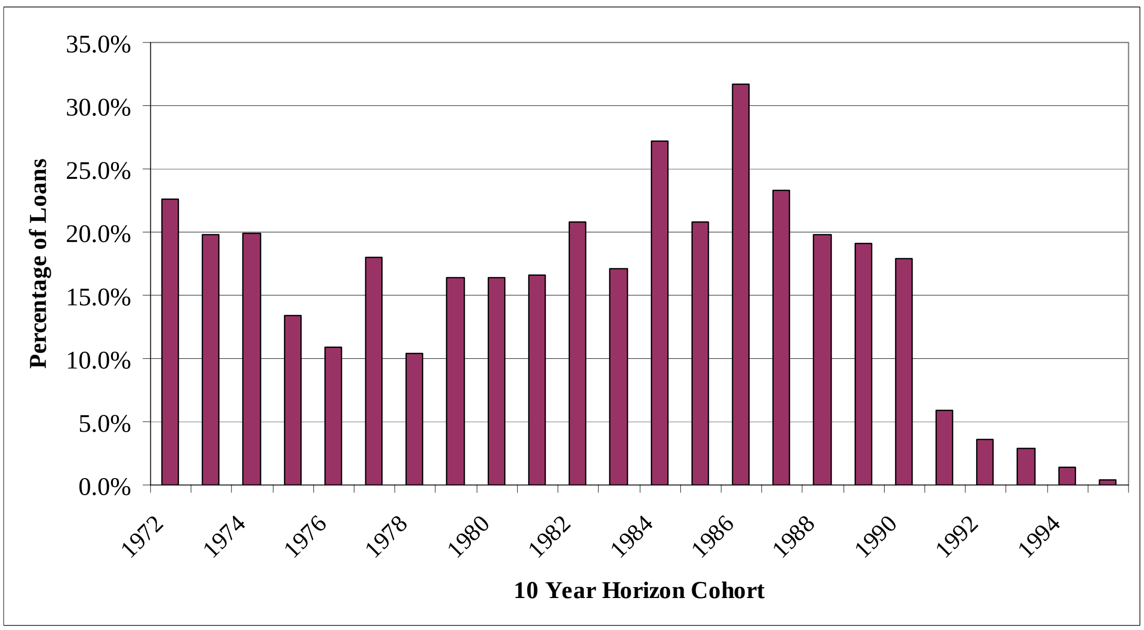


Figure 3: Liquidity Adjusted Implied Volatilities by Property Type

This Figure plots our calibrated liquidity adjusted implied volatilities by property type. The solid line plots the median implied volatility for mortgage originated within a quarter. The bottom dashed line plots the 25$^{\text{th}}$ quartile and the top dashed line plots the 75$^{\text{th}}$ quartile of the quarterly implied volatility distributions.

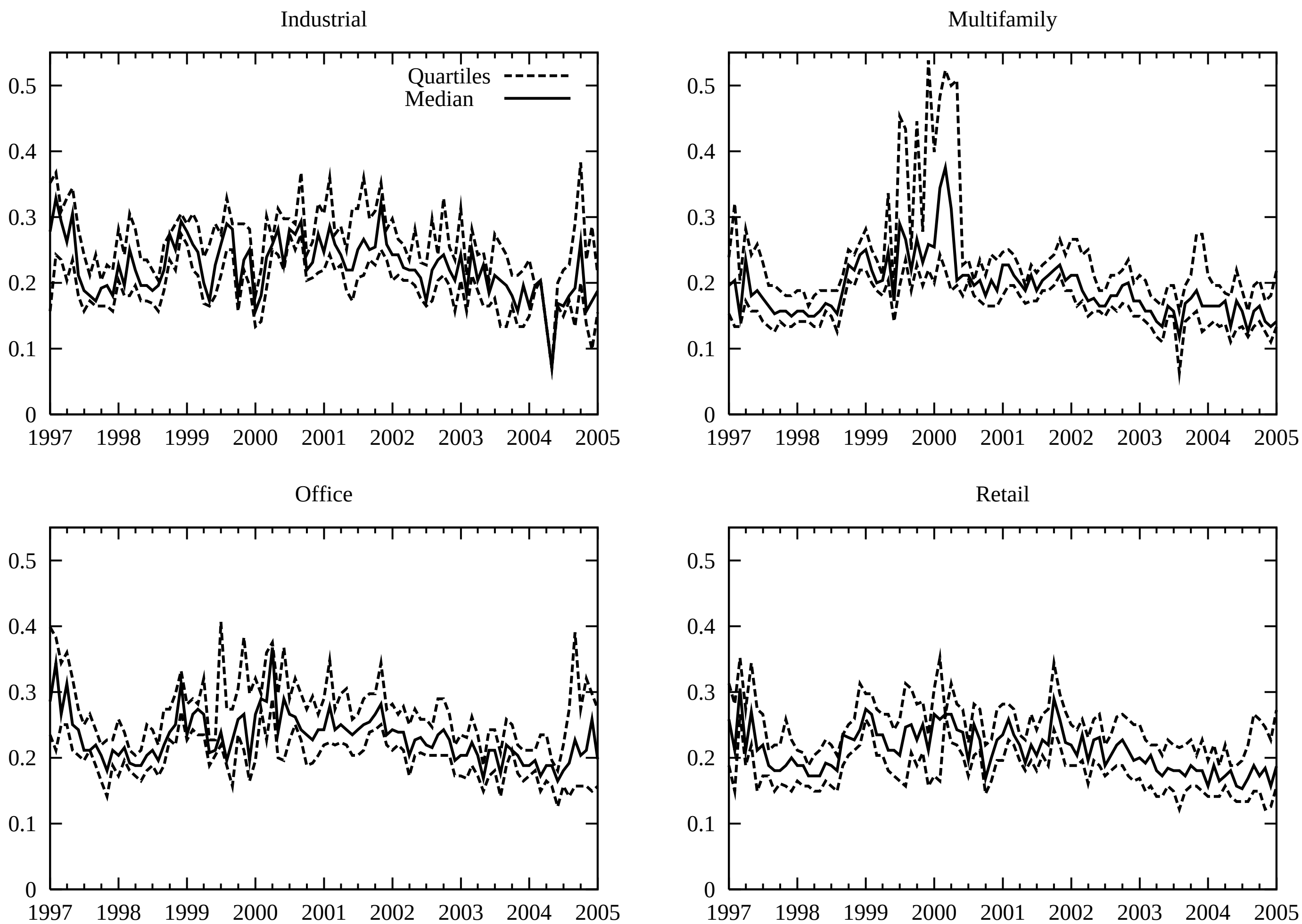
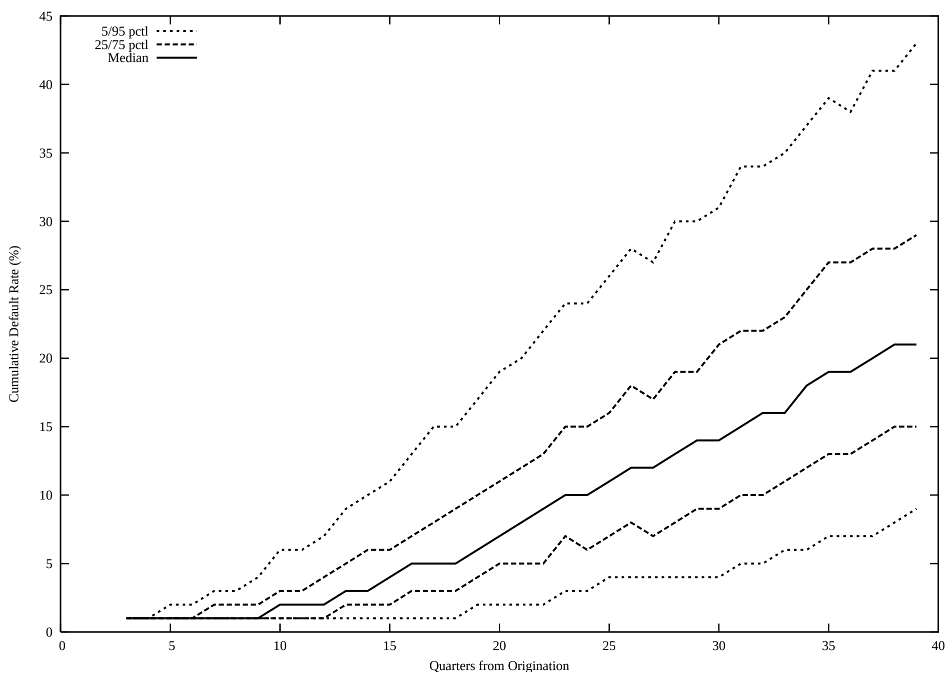


Figure 4: Simulated Cumulative Default Rates

This figures shows the distribution of cumulative default rates under our implied volatility measure. The solid line indicates the median cumulative default rate across the simulations, the dashed lines show the approximate location of the 25^{th} and 75^{th} percentiles, and the dotted lines show the 5^{th} and 95^{th} percentiles, respectively. We make 5,000 draws from the system of Equations (5) and (6) and keep track of the first time that each mortgage defaults along a simulated path of interest rates and property returns. For each LTV level and property type, we compute the default frequency by quarter. The weighted-average default frequency for the pool is computed using the property-type frequencies and the LTVs as weights. The cumulative default rates are computed by summing the quarterly default frequencies.



References

Ambrose, B., and A. Sanders, 2003, Commercial mortgage backed securities: Prepayment and default, *Journal of Real Estate Finance and Economics* 26, 175–192.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens, 2001a, The distribution of realized stock return volatility, *Journal of Financial Economics* 61, 43–76.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys, 2001b, The distribution of realized exchange rate volatility, *Journal of the American Statistical Association* 96, 42–55.

An, X., and Y. Deng, 2007, Subordination levels in commercial mortgage-backed securities (CMBS), Working paper, University of Southern California.

Archer, W., P. Elmer, D. Harrison, and D. Ling, 2002, Determinants of multifamily mortgage defaults, *Real Estate Economics* 30, 445–473.

Black, F., 1976, Studies of stock price volatility changes, in *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economic Statistic Section*, 177–181.

Brealey, R. A., S. C. Myers, and F. Allen, 2006, *Principles of Corporate Finance*, 8th edition (McGraw-Hill, New York, NY).

Britten-Jones, M., and A. Neuberger, 2000, Option prices, implied price processes, and stochastic volatility, *Journal of Finance* 55, 839–866.

Chapman, D. A., and N. D. Pearson, 2001, What can be learned from recent advances in estimating models of the term structure?, *Financial Analysts Journal* July/August, 77–95.

Childs, P. D., S. H. Ott, and T. J. Riddiough, 1996, The pricing of multi-class commercial mortgaged-backed securities, *Journal of Financial and Quantitative Analysis* 31, 581–603.

Ciochetti, B. A., and K. D. Vandell, 1999, The performance of commercial mortgages, *Real Estate Economics* 27, 27–61.

Ciochetti, B., Y. Deng, G. Lee, J. Shilling, and R. Yao, 2003, A proportional hazards model of commercial mortgage default with origination bias, *Journal of Real Estate Finance and Economics* 27, 5–23.

Clayton, J., D. Geltner, and S. Hamilton, 2001, Smoothing in commercial property valuations: Evidence from individual appraisals, *Real Estate Economics* 29, 337–360.

Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985, A theory of the term structure of interest rates, *Econometrica* 53, 386–407.

Dai, Q., and K. Singleton, 2003, Term structure dynamics in theory and reality, *Review of Financial Studies* 16, 631–678.

Downing, C., R. Stanton, and N. Wallace, 2005, An empirical test of a two-factor valuation model: How much do house prices matter?, *Real Estate Economics* 33, 681–710.

Dumas, B., J. Fleming, and R. Whaley, 1998, Implied volatility functions: Empirical tests, *Journal of Finance* 53, 2059–2106.

Esaki, H., 2002, Commercial mortgage defaults: 1972–2000, *Real Estate Finance* Winter, 43–52.

Esaki, H., 2003, Commercial mortgage defaults: 30 years of history, Fixed income research, Morgan Stanley.

Fisher, J., D. Gatzlaff, D. Geltner, and D. Haurin, 2003, Controlling for variable liquidity and selection bias in indices of private asset market values, Working paper, Indiana University.

Fisher, J., D. Geltner, and R. B. Webb, 1994, Value indexes of commercial real estate: A comparison of index construction methods, *Journal of Real Estate Finance and Economics* 9, 137–164.

Gatzlaff, D., and D. Geltner, 1998, A transaction-based index of commercial property and its comparison to the NCREIF index, *Real Estate Economics* 15, 7–22.

Geltner, D., and W. Goetzmann, 1998, Two decades of commercial property returns: A repeated-measures regression-based version of the NCREIF index, *Journal of Real Estate Finance and Economics* 32, 5–22.

Gourlay, A. R., and S. McKee, 1977, The construction of hopscotch methods for parabolic and elliptic equations in two space dimensions with a mixed derivative, *Journal of Computational and Applied Mathematics* 3, 201–206.

Holland, A. S., S. H. Ott, and T. J. Riddiough, 2000, The role of uncertainty in investment: An examination of competing investment models using commercial real estate data, *Real Estate Economics* 28, 33–64.

Jackwerth, J., and M. Rubinstein, 1996, Recovering probability distributions from option prices, *Journal of Finance* 51, 1611–1131.

Jarrow, C., and Y. Yildirim, 2006, Commercial mortgage backed securities (CMBS) and market efficiency with respect to costly information, Working paper, Syracuse University.

Jarrow, R., and S. Turnbull, 1995, Pricing derivatives on financial securities subject to default risk, *Journal of Finance* 50, 53–86.

Jiang, G. J., and Y. S. Tian, 2005, The model-free implied volatility and its information content, *Review of Financial Studies* 18, 1305–1342.

Johnson, B., and M. MacNeill, 2005, 2005 U.S. CMBS loss study, Commercial mortgage special report, FitchRatings.

Jorion, P., 1995, Predicting volatility in the foreign exchange market, *Journal of Finance* 50, 507–528.

Kau, J., D. C. Keenan, W. J. Muller, and J. F. Epperson, 1990, Pricing commercial mortgages and their mortgage backed securities, *Journal of Real Estate Finance and Economics* 3, 333–356.

McConnell, J. J., and M. K. Singh, 1994, Rational prepayments and the valuation of collateralized mortgage obligations, *Journal of Finance* 49, 891–921.

Merton, R. C., 1980, On estimating the expected return on the market, *Journal of Financial Economics* 8, 323–361.

O'Rourke, M., J. Story, and A. Zaffuto, 2005, 2005 CMBS conduit loan default study, FitchRatings: Commercial Mortgage Special Report.

Pearson, N. D., and T.-S. Sun, 1989, A test of the Cox, Ingersoll, Ross model of the term structure of interest rates using the method of maximum likelihood, Working paper, MIT.

Pong, S., M. B. Shackleton, S. J. Taylor, and X. Xu, 2004, Forecasting sterling/dollar volatility: A comparison of implied volatility and AR(FI)MA models, *Journal of Banking and Finance* 28, 2541–2563.

Quigg, L., 1993, Empirical testing of real option-pricing models, *Journal of Finance* 48, 621–640.

Schwert, G. W., 1989, Why does stock market volatility change over time?, *Journal of Finance* 44, 1115–1154.

Schwert, G. W., 1990, Stock volatility and the crash of '87, *Review of Financial Studies* 3, 77–102.

Seslen, T., and W. Wheaton, 2005, Contemporaneous loan stress and termination risk in the CMBS pool: How "ruthless" is default?, Working paper, University of Southern California.

Snyderman, M., 1991, Commercial mortgages: Default occurrence and estimated yield impact, *Journal of Portfolio Management* 18, 82–87.

Synderman, M., 1994, Update on commercial mortgage default, *Real Estate Economics* 10, 22–32.

Titman, S., S. Tompaidis, and S. Tsyplakov, 2004, Market imperfections, investment flexibility and default spreads, *Journal of Finance* 59, 165–206.

Titman, S., and W. Torous, 1989, Valuing commercial mortgages: An empirical investigation of the contingent claims approach to pricing risky debt, *Journal of Finance* 44, 345–373.

Tomlinson, R., and D. Evans, 2007, CDO boom masks subprime losses, abetted by S&P, Moody's, Fitch, Bloomberg.com.

Tu, Y., S. M. Yu, and H. Sun, 2004, Transaction-based office price indexes: A spatiotemporal modeling approach, *Real Estate Economics* 32, 297–328.

Vandell, K., W. Barnes, D. Hartzell, D. Kraft, and W. Wendt, 1993, Commercial mortgage defaults: Proportional hazards estimation using individual loan histories, *Real Estate Economics* 21, 451–480.

Wheeler, D., 2001, A guide to commercial mortgage-backed securities, in L. Hayre, ed., *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*, chapter 16, 399–451 (John Wiley, New York).

Zhou, B., 1996, High-frequency data and volatility in foreign-exchange rates, *Journal of Business and Economic Statistics* 14, 45–52.