Asset-level risk and return in real estate investments

Jacob S. Sagi

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Individual real estate assets

- Highly illiquid
  - Trade infrequently
  - Costly to trade
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- Large market
  - Investment market: ~ $3T as of 2010 (Geltner et al., 2013)
Individual real estate assets

- Highly illiquid
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  - Costly to trade
- Large market
  - Investment market: ~ $3T as of 2010 (Geltner et al., 2013)
- Idiosyncratic risk matters
  - A third of investment-grade RE held privately (Geltner et al., 2013)
  - RE investments underwritten at asset-level
  - Secured debt
  - CMBS
What do we know?

- CMBS (Downing et al., 2008)
  - Back out implied volatility from loans
  - 20%-24% idiosyncratic vol
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- Peng (2014)
  - Same data set as this paper
  - Examines systematic exposure of property-level returns
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- **Plazzi, Torous and Valkanov (2008)**
  - Geographic-level data from Global Real Analytics
  - 4%-7% dispersion in means
  - Much of the property-specific risk is diversified
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All assume random walk hypothesis.
Idiosyncratic h.p.l.p.a. mean and variance

Estimate $a_\tau$ and $\sigma^2_\tau$ in

$$r^\text{App,e}_{i,\tau} = a_\tau + \beta r^e_{m,\tau} + \sigma_\tau \varepsilon_i.$$
Results: Holding period residual return variance

<table>
<thead>
<tr>
<th>Holding Period (years)</th>
<th>Variance of Market-adjusted Log-Price Appreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
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<tr>
<td>2</td>
<td>0.2</td>
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<table>
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<tr>
<th>Coeff</th>
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</table>

Sagi, Jacob S.  Asset-level risk and return in real estate investments
Results: Holding period idiosyncratic return mean

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>-1.6</td>
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<td>3</td>
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<th>coeff</th>
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<tr>
<td>( \tau )</td>
<td>-0.0871</td>
<td>-24.7</td>
<td>0.990</td>
<td>14</td>
</tr>
<tr>
<td>Const</td>
<td>0.0757</td>
<td>3.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Extrapolated Sharpe Ratio for zero holding period:
$$\frac{0.076}{\sqrt{0.041}} \approx 0.37.$$
The puzzle

- Slopes of graphs are consistent with constant drift & volatility of random walk process.$^a$

- Intercepts of graphs imply infinite drift & volatility as holding period $\to 0$.

$^a$Not with CMBS implied property volatility in (Downing et al., 2008).
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  - Vintage effects
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- Results arise from illiquidity of the underlying
  - Equilibrium search model with random matching and bargaining
  - Gains from trade come from persistent but heterogeneous private valuations of property income streams
  - Qualitatively reproduce results
  - Can be calibrated to the data
An explanation?

- Can’t arbitrage off zero holding period extrapolated Sharpe Ratio ($\approx 0.37$).
  - Real estate is too illiquid to hold for very short periods.
An explanation?

- Can’t arbitrage off zero holding period extrapolated Sharpe Ratio ($\approx 0.37$).
  - Real estate is too illiquid to hold for very short periods
- Can illiquidity explain the empirical finding?

- Large number of infinitely-lived investors and income-producing properties
An equilibrium random search and bargaining model


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- Property income is $d_t$
An equilibrium random search and bargaining model


- Large number of infinitely-lived investors and income-producing properties
- Property income is $d_t$
- Investor $i$ discounts next period’s payoffs with rate $r_{i,t} \in A$
  - $A$ is a finite set of discount rate “types”
  - $r_{i,t}$ is Markov process with transition matrix $\Pi^T_{aa'}$ representing probability of going from type $a$ to $a'$
  - $r_{i,t}$ independent across investors and not correlated with income shocks
An equilibrium random search and bargaining model

Assumptions:

- Owners receive one bid per period from investor drawn from the unconditional distribution of types, $\pi^U$.
- Investors can own more than one property
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Assumptions:

- Owners receive one bid per period from investor drawn from the unconditional distribution of types, $\pi^U$.
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- Random relative bargaining power (gains from trade are allocated randomly)
- Seller pays transaction cost of $c_t$
Property owned by type $a$

Owner receives offer from type $a'$

Owner's type transitions

$t$

Offer rejected or accepted & ownership is transferred

$t+1$
Private value of ownership

\[ p_t(r_{i,t}) = \frac{1}{1 + r_{i,t}} E \left[ \tilde{d}_{t+1} + p_{t+1}(\tilde{r}_{i,t+1}) \right. \]
\[ + \tilde{\lambda} \left\{ p_{t+1}(\tilde{r}^{'}) - p_{t+1}(\tilde{r}_{i,t+1}) - \tilde{c}_{t+1} \right\}^+ \]...

- \( \{x\}^+ \equiv \max\{0, x\} \)
- \( \tilde{r}_{i,t+1} \) and \( \tilde{r}^{'} \) are seller's and buyer's random discount rates at date \( t + 1 \) (independent)
- \( \tilde{\lambda} \) is random allocation of gains from trade (relative bargaining power) — independent of other RVs
Definitions

**Equilibrium**

An equilibrium is a positive and finite random variable $p_t(r_a)$ that solves (1) for every $a \in \mathcal{A}$. 
Definitions

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<th>Equilibrium steady state</th>
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<td>A steady state is achieved once the distribution of ownership across properties is not expected to change.</td>
</tr>
</tbody>
</table>
Useful detour: The frictionless ideal

If $c_t = 0$ and number of bids can be arbitrarily large,

$$p_t(r_{i,t}) = \frac{1}{1 + r_{i,t}} E \left[ \tilde{d}_{t+1} + p_{t+1}(\tilde{r}_{i,t+1}) \right] + \max \left\{ 0, \tilde{\lambda}'(p_{t+1}(\tilde{r}')) - p_{t+1}(\tilde{r}_{i,t+1}), \tilde{\lambda}''(p_{t+1}(\tilde{r}'') - p_{t+1}(\tilde{r}_{i,t+1}), \tilde{\lambda}'''(p_{t+1}(\tilde{r}''') - p_{t+1}(\tilde{r}_{i,t+1}), \ldots \right\}^+.$$
If $c_t = 0$ and number of bids can be arbitrarily large,

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Bidder types are dense in support of $\tilde{\lambda}$ and $\tilde{r} \Rightarrow$ winning bid:
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Bidder types are dense in support of $\tilde{\lambda}$ and $\tilde{r} \Rightarrow$ winning bid:

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Bidder types are dense in support of $\tilde{\lambda}$ and $\tilde{r}$ ⇒ winning bid:

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$$\tilde{\lambda}''(p_{t+1}(\tilde{r}'') - p_{t+1}(\tilde{r}_{i,t+1})), \tilde{\lambda}'''(p_{t+1}(\tilde{r}''') - p_{t+1}(\tilde{r}_{i,t+1})), \ldots \left\}^+ \right].$$

Bidder types are dense in support of $\tilde{\lambda}$ and $\tilde{r} \Rightarrow$ winning bid:

- least bargaining power (highest $\lambda$)
- highest valuation

Steady state owners will be highest valuation types and

$$p_t = \frac{E[\tilde{d}_{t+1} + \tilde{p}_{t+1}]}{1 + r}, \text{ where } r \text{ is smallest rate.}$$
Random walk income model

- $d_{t+1} = d_t e^{\mu - \frac{\sigma^2}{2}} + \sigma \tilde{\epsilon}_{t+1}$
- $c_t = cd_t$
Random walk income model

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Conjecture equilibrium where

$$p_t(r_a) = d_t Q_a.$$
Random walk income model

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- \( c_t = cd_t \)

Conjecture equilibrium where

\[ p_t(r_a) = d_t Q_a. \]

Then \( Q_a \) solves linear set of equations:

\[
\forall a \in \mathcal{A}, \quad (1 + r_a)e^{-\mu}Q_a = (1 + \sum_{a' \in \mathcal{A}} \Pi_{a a'}^T Q_{a'}) + \bar{\lambda} \sum_{a', b \in \mathcal{A}} \Pi_{a a'}^T \pi_b \left\{ Q_b - Q_{a'} - c \right\}^+. 
\]
Transaction prices

Transaction can only take place if $Q_b - Q_a \geq c$

- $Q_b$ is buyer’s valuation
- $Q_a$ is seller’s valuation
Transaction prices

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Observed (net) transaction prices

$$P_{t,ab} = d_t \left( Q_a + \tilde{\lambda}(Q_b - Q_a - c) \right), \quad \text{s.t.} \quad Q_b - Q_a' \geq c$$
Holding period returns

- At date $t$, property is bought in the steady state by type $Q_{i,t}$ from type $O$
Holding period returns

- At date $t$, property is bought in the steady state by type $Q_{i,t}$ from type $O$
- New owner receives offers for $\tau - 1$ periods but does not accept
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- At date $\tau$, the owner (now type $Q_{i,t+\tau}$) accepts offer from type $S$
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Holding period price appreciation returns

$$\tilde{R}_{i,t,\tau} = \frac{Q_{i,t+\tau} + \tilde{\lambda}'(\tilde{Q}_S - Q_{i,t+\tau} - c)}{\tilde{Q}_O + c + \tilde{\lambda}(Q_{i,t} - \tilde{Q}_O - c)} e^{(\mu - \frac{\sigma^2}{2})\tau + \sigma \sqrt{\tau}\tilde{n}},$$

$\tilde{n}$ is standard normal, $\tilde{\lambda}$ and $\tilde{\lambda}'$ are iid (note denominator).
Holding period idiosyncratic log returns

Let $r_m$ be mean market return (assume normal) and $\sigma_I$ be property idiosyncratic vol. Assume property $\beta_i = 1$. 
Holding period idiosyncratic log returns

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\[
\ln \tilde{R}_{i,t} = \ln \left( Q_{i,t} + \tilde{\lambda}'(\tilde{Q}_s - Q_{i,t} - c) \right)
\]

\[
- \ln \left( \tilde{Q}_O + c + \tilde{\lambda}(Q_{i,t} - \tilde{Q}_O - c) \right)
\]

\[
+ \sigma_I \sqrt{\tau} \tilde{n} + (\mu - \frac{\sigma_I^2}{2} - r_m)\tau.
\]

conditional on observing the purchase and sale....
Holding period idiosyncratic log returns

Let $r_m$ be mean market return (assume normal) and $\sigma_I$ be property idiosyncratic vol. Assume property $\beta_i = 1$.

$$\ln \tilde{R}_{i,t,\tau} = \ln \left( Q_{i,t+\tau} + \tilde{\lambda}'(\tilde{Q}_S - Q_{i,t+\tau} - c) \right)$$

Selling shock

$$- \ln \left( \tilde{Q}_O + c + \tilde{\lambda}(Q_{i,t} - \tilde{Q}_O - c) \right)$$

Purchasing shock

$$+ \sigma_I \sqrt{\tau} \tilde{n} + (\mu - \frac{\sigma_I^2}{2} - r_m)\tau.$$

Income shock

conditional on observing the purchase and sale....

- Selling and purchasing shocks are idiosyncratic
Holding period log returns: IID vs. Persistent Types

- Type transition cannot be completely random
Holding period log returns: IID vs. Persistent Types

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- What about persistent types?
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- Type transition cannot be completely random
- What about persistent types?
  - New buyer will be “high valuation” type
  - Unlikely to change types the following period
  - Likely to sell only if bid is higher than private value
    - And therefore greater than last period’s purchase price
  - Short holding periods correspond to “better than purchase price offers” and positive expected returns
A simple calibration

- Three states: $r_1 < r_2 < r_3$
A simple calibration

- Three states: \( r_1 < r_2 < r_3 \)
- Set \( x, y > 0 \) and \( x + y < 1 \). Let

\[
\Pi_{aa'}^T = \begin{pmatrix}
1 - x - y & x & y \\
x & 1 - 2x & x \\
y & x & 1 - x - y
\end{pmatrix}
\]
A simple calibration

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$$\Pi^T_{a a'} = \begin{pmatrix} 1 - x - y & x & y \\ x & 1 - 2x & x \\ y & x & 1 - x - y \end{pmatrix}$$

- $$\pi^U = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^T$$
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- Also determines the distribution of $\tilde{Q}_S$ in $\ln \tilde{R}_{i,t,\tau}$. 

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- $\pi^U = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^T$
  - Also determines the distribution of $\tilde{Q}_S$ in $\ln \tilde{R}_{i,t,\tau}$.
- Steady state distribution ($\tilde{Q}_O$ in $\ln \tilde{R}_{i,t,\tau}$) given by

$$
\pi^O = \left(\frac{4x - y + 2}{6x^2 + 2x(6y + 5) + 5y + 2}, \frac{3(2x + y)}{6x^2 + 2x(6y + 5) + 5y + 2}, \frac{3\left(2x^2 + 4xy + y\right)}{6x^2 + 2x(6y + 5) + 5y + 2}\right)^T.
$$
Simple calibration, cont.

For the $\tau - 1$ “survival” probability, see paper. In calibration, set

For the $\tau - 1$ “survival” probability, see paper. In calibration, set
Simple calibration, cont.

For the $\tau - 1$ “survival” probability, see paper. In calibration, set

- Quarterly periods
- $r_1 = 0.0175$, $r_2 = 0.075$ and $r_3 = 0.125$ set to match time-independent variance in baseline calibration
- Idiosyncratic property volatility is set via $\sigma_i^2 = 0.0108$
- Property market return $r_M$ is set to 10%
- $c$ and income growth rate $\mu$ set to match median sales cost (0.0212) and mean annualized cap rates (0.0690)
- $x = y$ varied and equal 0.015 in baseline calibration
## Calibration: Model comparison

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimated $2.5^{th}$ percentile</th>
<th>Estimated $97.5^{th}$ percentile</th>
<th>Calibrated Model</th>
<th>Calibrated Model 2</th>
<th>Calibrated Model 3</th>
<th>Calibrated Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = y$ (Qtrly)</td>
<td>NA</td>
<td>NA</td>
<td>0.013</td>
<td>0.030</td>
<td>0.050</td>
<td>0.100</td>
</tr>
<tr>
<td>$c$</td>
<td>NA</td>
<td>NA</td>
<td>1.228</td>
<td>1.228</td>
<td>1.228</td>
<td>1.228</td>
</tr>
<tr>
<td>$\mu$ (Qtrly)</td>
<td>-0.0022</td>
<td>0.0141</td>
<td>0.0082</td>
<td>0.0140</td>
<td>0.0193</td>
<td>0.0282</td>
</tr>
<tr>
<td>1yr Holding Per.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. Exp. Return</td>
<td>-0.038</td>
<td>0.024</td>
<td>-0.021</td>
<td>-0.043</td>
<td>-0.041</td>
<td>-0.017</td>
</tr>
<tr>
<td>Adj. Variance</td>
<td>0.048</td>
<td>0.060</td>
<td>0.051</td>
<td>0.044</td>
<td>0.036</td>
<td>0.024</td>
</tr>
<tr>
<td>Fraction Sold</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>After 6.25 Years</td>
<td>0.491</td>
<td>0.508</td>
<td>0.497</td>
<td>0.790</td>
<td>0.922</td>
<td>0.991</td>
</tr>
<tr>
<td>Quarterly Turnover</td>
<td>NA</td>
<td>NA</td>
<td>0.030</td>
<td>0.065</td>
<td>0.100</td>
<td>0.168</td>
</tr>
<tr>
<td>Liquidity Premium</td>
<td>NA</td>
<td>NA</td>
<td>0.114</td>
<td>0.084</td>
<td>0.059</td>
<td>0.021</td>
</tr>
<tr>
<td>Fire Sale Discount</td>
<td>NA</td>
<td>NA</td>
<td>0.210</td>
<td>0.188</td>
<td>0.167</td>
<td>0.131</td>
</tr>
</tbody>
</table>
Calibration: Model comparison

Market-adjusted Expected Log-Price Appreciation

Holding Period (years)

- Actual
- x=0.013
- x=0.03
- x=0.05
- x=0.1

Sagi, Jacob S. Asset-level risk and return in real estate investments
Calibration: Fit of baseline model
Calibration: Fit of baseline model

Introduction
An empirical puzzle
Model
Empirical robustness
Conclusion

Sagi, Jacob S. Asset-level risk and return in real estate investments
Can we go home now?
Can we go home now?

No. Empirical results on holding period returns may be spurious

- Vintage effects
Can we go home now?

No. Empirical results on holding period returns may be spurious

- Vintage effects
- Random coefficients
Can we go home now?

No. Empirical results on holding period returns may be spurious

- Vintage effects
- Random coefficients
- Selection bias 1: Safer assets held longer
Can we go home now?

No. Empirical results on holding period returns may be spurious

- Vintage effects
- Random coefficients
- Selection bias 1: Safer assets held longer
- Selection bias 2: Optimal disposition
## Vintage effects

### Table 2: The histogram reports entries of properties into the database (i.e., “vintages”). The table reports the average vintage for various holding periods.

<table>
<thead>
<tr>
<th>Holding Period</th>
<th>Average Vintage</th>
<th>Number of Props</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2002</td>
<td>724</td>
</tr>
<tr>
<td>2</td>
<td>2001</td>
<td>770</td>
</tr>
<tr>
<td>3</td>
<td>2001</td>
<td>905</td>
</tr>
<tr>
<td>4</td>
<td>2000</td>
<td>731</td>
</tr>
<tr>
<td>5</td>
<td>2000</td>
<td>700</td>
</tr>
<tr>
<td>6</td>
<td>1997</td>
<td>593</td>
</tr>
<tr>
<td>7</td>
<td>1997</td>
<td>589</td>
</tr>
<tr>
<td>8</td>
<td>1994</td>
<td>477</td>
</tr>
<tr>
<td>9</td>
<td>1991</td>
<td>277</td>
</tr>
<tr>
<td>10</td>
<td>1990</td>
<td>236</td>
</tr>
<tr>
<td>11</td>
<td>1990</td>
<td>174</td>
</tr>
<tr>
<td>12</td>
<td>1990</td>
<td>125</td>
</tr>
<tr>
<td>13</td>
<td>1988</td>
<td>108</td>
</tr>
<tr>
<td>14</td>
<td>1987</td>
<td>75</td>
</tr>
<tr>
<td>15</td>
<td>1986</td>
<td>63</td>
</tr>
<tr>
<td>16</td>
<td>1983</td>
<td>34</td>
</tr>
</tbody>
</table>
Controlling for vintage effects: Strategy

- Randomly match property held between $t$ and $t + k \times \tau$ with $k$ properties respectively held between $t$ and $t + \tau$, $t$ and $t + 2\tau$, etc.
Controlling for vintage effects: Strategy

- Randomly match property held between $t$ and $t + k \times \tau$ with $k$ properties respectively held between $t$ and $t + \tau$, $t$ and $t + 2\tau$, etc.
  - Like a portfolio “roll-over” strategy
Controlling for vintage effects: Strategy

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  - Like a portfolio “roll-over” strategy
- Use a linear program to maximize number of such matches
Controlling for vintage effects: Strategy

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- Compare matched idiosyncratic mean and variance
Controlling for vintage effects: Strategy

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  - Like a portfolio “roll-over” strategy

- Use a linear program to maximize number of such matches

- Compare matched idiosyncratic mean and variance
  - Repeat 100 times and average stats to ensure all of data is utilized
Controlling for vintage effects: Strategy

- Randomly match property held between \( t \) and \( t + k \times \tau \) with \( k \) properties respectively held between \( t \) and \( t + \tau \), \( t \) and \( t + 2\tau \), etc.
  - Like a portfolio “roll-over” strategy
- Use a linear program to maximize number of such matches
- Compare matched idiosyncratic mean and variance
  - Repeat 100 times and average stats to ensure all of data is utilized
  - Standard error of estimate does not incorporate the repetition (conservative)
### Vintage effects: Results

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\tau$</th>
<th>$N$</th>
<th>Avg Vintage</th>
<th>$(\text{Var Diff})/(k - 1)$</th>
<th>$t$-stat</th>
<th>$(\alpha \text{ Diff})/(k - 1)$</th>
<th>$t$-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>321</td>
<td>2002</td>
<td>0.056</td>
<td>5.37</td>
<td>-0.004</td>
<td>-0.09</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>315</td>
<td>2000</td>
<td>0.054</td>
<td>5.14</td>
<td>0.099</td>
<td>2.27</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>240</td>
<td>1999</td>
<td>0.056</td>
<td>4.30</td>
<td>0.068</td>
<td>1.27</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>237</td>
<td>1998</td>
<td>0.033</td>
<td>2.61</td>
<td>-0.023</td>
<td>-0.37</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>126</td>
<td>1997</td>
<td>0.029</td>
<td>2.59</td>
<td>0.095</td>
<td>2.08</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>140</td>
<td>1994</td>
<td>0.022</td>
<td>1.23</td>
<td>0.008</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>76</td>
<td>1993</td>
<td>0.033</td>
<td>1.24</td>
<td>-0.055</td>
<td>-0.58</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>58</td>
<td>1988</td>
<td>0.003</td>
<td>0.11</td>
<td>-0.183</td>
<td>-2.23</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>39</td>
<td>1988</td>
<td>0.015</td>
<td>1.06</td>
<td>0.020</td>
<td>0.43</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>31</td>
<td>1982</td>
<td>0.134</td>
<td>2.49</td>
<td>0.067</td>
<td>0.59</td>
</tr>
</tbody>
</table>

**Prob $\chi(\text{Var Diff}= 0)$**: 4.7E-17

**Prob $\chi(\alpha = 0)$**: 0.0474

<table>
<thead>
<tr>
<th>Statistic</th>
<th>GLS Estimate</th>
<th>se</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var Diff</td>
<td>0.0404</td>
<td>0.0060</td>
</tr>
</tbody>
</table>
Random coefficients

\[
R_{it}^{App,e} = a_i - (\delta_i + \xi_i)\tau + \beta_i r_{it}^e + \sigma_i \sqrt{\tau} \varepsilon_i
\]

Heterogeneity of properties coefficients can lead to mis-measured estimates of idiosyncratic mean and variance as functions of \(\tau\).

- Estimate random effects model

\[
\tilde{R}_{it}^{App,e} = \alpha_0 \frac{1}{\sqrt{\tau}} + \alpha_1 \sqrt{\tau} + \beta \frac{r_{it}^e}{\sqrt{\tau}} + (\tilde{\varepsilon}_0 \frac{1}{\sqrt{\tau}} + \tilde{\varepsilon}_1 \sqrt{\tau} + \tilde{\varepsilon}_\beta \frac{r_{it}^e}{\sqrt{\tau}} + \sigma \tilde{\varepsilon})
\]
Random coefficients

\[ r_{i}^{\text{App,e}} = a_{i} - (\delta_{i} + \xi_{i})\tau + \beta_{i}r_{m}^{e} + \sigma_{i}\sqrt{\tau}\varepsilon_{i} \]

Heterogeneity of properties coefficients can lead to mis-measured estimates of idiosyncratic mean and variance as functions of \( \tau \).

- Estimate random effects model

\[ \tilde{r}_{\text{App,e}}^{\tau} = \alpha_{0} \frac{1}{\sqrt{\tau}} + \alpha_{1} \sqrt{\tau} + \beta \frac{r_{m}^{e}}{\sqrt{\tau}} + (\tilde{\varepsilon}_{0} \frac{1}{\sqrt{\tau}} + \tilde{\varepsilon}_{1} \sqrt{\tau} + \tilde{\varepsilon}_{\beta} \frac{r_{m}^{e}}{\sqrt{\tau}} + \sigma_{\tilde{\varepsilon}}) \]

- Want to back out \( \alpha_{0}, \alpha_{1}, \sigma_{0}^{2} \equiv \text{VAR[\tilde{\varepsilon}_{0}]} \) and \( \sigma^{2} \equiv \text{VAR[\tilde{\varepsilon}]} \)

- Four-pass regression (Hildreth and Houck, 1968; Swamy, 1970; Raj, Srivastava, and Ullah, 1980)
Random effects: Results

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{r}_{\text{App},e}^{2}$</td>
<td>0.960</td>
<td>0.946</td>
</tr>
<tr>
<td>Adj $\hat{r}_{\text{App},e}^{2}$</td>
<td>0.946</td>
<td>0.946</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.960</td>
<td>0.946</td>
</tr>
<tr>
<td></td>
<td>(48.80)</td>
<td>(52.34)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-0.0875</td>
<td>-0.0864</td>
</tr>
<tr>
<td></td>
<td>(-63.62)</td>
<td>(-65.20)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.0787</td>
<td>0.0748</td>
</tr>
<tr>
<td></td>
<td>(12.37)</td>
<td>(10.54)</td>
</tr>
<tr>
<td>Observations</td>
<td>6287</td>
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</tr>
</tbody>
</table>
## Random effects: Results

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{z}^2 )</td>
<td>-0.0000736</td>
<td>-0.0000900</td>
<td>0.000242</td>
<td>0.000184</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.13)</td>
<td>(-0.16)</td>
<td>(1.01)</td>
<td>(0.79)</td>
<td></td>
</tr>
<tr>
<td>( 2\sigma_1^2 \beta )</td>
<td>-0.00445</td>
<td>-0.00443</td>
<td>-0.00421</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.18)</td>
<td>(-1.18)</td>
<td>(-1.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_0^2 )</td>
<td>-0.0749</td>
<td>-0.0755</td>
<td>-0.0791</td>
<td>-0.118</td>
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</tr>
<tr>
<td></td>
<td>(-1.62)</td>
<td>(-1.63)</td>
<td>(-1.72)</td>
<td>(-3.89)</td>
<td></td>
</tr>
<tr>
<td>( \sigma_0^2 )</td>
<td>0.0618</td>
<td>0.0616</td>
<td>0.0419</td>
<td>0.0421</td>
<td>0.0404</td>
</tr>
<tr>
<td></td>
<td>(2.05)</td>
<td>(2.04)</td>
<td>(10.17)</td>
<td>(10.21)</td>
<td>(14.56)</td>
</tr>
<tr>
<td>( 2\sigma_0r )</td>
<td>-0.0290</td>
<td>-0.0289</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.66)</td>
<td>(-0.66)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.0227</td>
<td>0.0228</td>
<td>0.0121</td>
<td>0.0122</td>
<td>0.0113</td>
</tr>
<tr>
<td></td>
<td>(1.38)</td>
<td>(1.38)</td>
<td>(4.81)</td>
<td>(4.85)</td>
<td>(11.72)</td>
</tr>
<tr>
<td>Collinearity</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Observations</td>
<td>6287</td>
<td>6287</td>
<td>6287</td>
<td>6287</td>
<td>6287</td>
</tr>
</tbody>
</table>
Comparison with “naive” estimates

Not significantly different!
Endogeneity I: Risk-horizon preferences

Is it possible that investors have a preference for holding less volatile properties over longer periods?
Endogeneity I: Risk-horizon preferences

Is it possible that investors have a preference for holding less volatile properties over longer periods?

- If true, then property risk characteristics should predict propensity to sell
- Run Logit on panel
  - Dependent variable is one at quarter $t$ if property is sold between $t + 1$ and $t + 4$
  - Independent variables are property-specific characteristics related to property risk profile
## Logit: Sale versus risk characteristics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff</th>
<th>t-stat</th>
<th>Marginal impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>SqFt</td>
<td>-1.08E-07</td>
<td>-4.47</td>
<td>-0.0034</td>
</tr>
<tr>
<td>JV</td>
<td>0.1060</td>
<td>5.43</td>
<td>0.0075</td>
</tr>
<tr>
<td>Age when acquired</td>
<td>0.0061</td>
<td>11.35</td>
<td>0.0119</td>
</tr>
<tr>
<td>Percent Leased</td>
<td>-0.5002</td>
<td>-8.75</td>
<td>-0.0085</td>
</tr>
<tr>
<td>Loan spread</td>
<td>2.2742</td>
<td>2.65</td>
<td>0.0005</td>
</tr>
<tr>
<td><strong>Apartments</strong></td>
<td>0.5007</td>
<td>20.49</td>
<td><strong>0.0394</strong></td>
</tr>
<tr>
<td>Industrial</td>
<td>-0.0430</td>
<td>-1.76</td>
<td>-0.0030</td>
</tr>
<tr>
<td>Office</td>
<td>0.1705</td>
<td>6.83</td>
<td>0.0123</td>
</tr>
<tr>
<td>East</td>
<td>0.0527</td>
<td>2.50</td>
<td>0.0037</td>
</tr>
<tr>
<td>Midwest</td>
<td>-0.0093</td>
<td>-0.39</td>
<td>-0.0006</td>
</tr>
<tr>
<td>South</td>
<td>0.1723</td>
<td>8.71</td>
<td>0.0124</td>
</tr>
<tr>
<td>Lagged return</td>
<td>-0.7748</td>
<td>-20.05</td>
<td>-0.0272</td>
</tr>
<tr>
<td><strong>Idiosyncratic variance</strong></td>
<td>-6.9031</td>
<td>-13.50</td>
<td><strong>-0.0135</strong></td>
</tr>
<tr>
<td>$R^2_a$</td>
<td>-1.7288</td>
<td>-38.69</td>
<td>-0.0950</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.0145</td>
<td>9.23</td>
<td>0.0043</td>
</tr>
<tr>
<td>Mgr Type</td>
<td>0.5835</td>
<td>20.09</td>
<td>0.0339</td>
</tr>
<tr>
<td>Const</td>
<td>-6.154</td>
<td>-14.61</td>
<td></td>
</tr>
</tbody>
</table>

Observations: 228,935
Endogeneity I: Mixed results for risk-horizon preferences

- Cumulative impact of unsupportive variables is greater
Endogeneity I: Mixed results for risk-horizon preferences

- Cumulative impact of unsupportive variables is greater
- IVar of appraisal-based returns is unsupportive of hypothesis
Endogeneity I: Mixed results for risk-horizon preferences

- Cumulative impact of unsupportive variables is greater
- IVar of appraisal-based returns is unsupportive of hypothesis
- Most impactful variables related to different kind of “risk”
Endogeneity I: Mixed results for risk-horizon preferences

- Cumulative impact of unsupportive variables is greater
- IVar of appraisal-based returns is unsupportive of hypothesis
- Most impactful variables related to different kind of “risk”
- Lagged returns important
  - Suggests different type of endogeneity might be at play
Endogeneity II: Performance-based selection bias

Logit provides evidence that underperforming properties are more likely to be sold
Endogeneity II: Performance-based selection bias

Logit provides evidence that underperforming properties are more likely to be sold

- Consistent with optimal disposition
  - Property $\alpha_t$ is not known at purchase
Endogeneity II: Performance-based selection bias

Logit provides evidence that underperforming properties are more likely to be sold

- Consistent with optimal disposition
  - Property $\alpha_t$ is not known at purchase
  - When purchased, $\alpha_0 > 0$
Endogeneity II: Performance-based selection bias

Logit provides evidence that underperforming properties are more likely to be sold

- Consistent with optimal disposition
  - Property $\alpha_t$ is not known at purchase
  - When purchased, $\alpha_0 > 0$
  - Manager learns about $\alpha_t$ from property’s market adjusted performance
Endogeneity II: Performance-based selection bias

Logit provides evidence that underperforming properties are more likely to be sold

- Consistent with optimal disposition
  - Property $\alpha_t$ is not known at purchase
  - When purchased, $\alpha_0 > 0$
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  - If $\alpha_t$ is below some threshold, property is sold
Endogeneity II: Performance-based selection bias

Logit provides evidence that underperforming properties are more likely to be sold

- Consistent with optimal disposition
  - Property $\alpha_t$ is not known at purchase
  - When purchased, $\alpha_0 > 0$
  - Manager learns about $\alpha_t$ from property’s market adjusted performance
  - If $\alpha_t$ is below some threshold, property is sold

- Paths of properties with observed transactions is not representative and may understate true variance

- Model this to explore empirical predictions
Model of optimal disposition

\[ dr_t = dr_t^O + dr_t^U \quad r^O \text{ observed }, r^U \text{ unobserved/unrelated} \]
Model of optimal disposition

\[ dr_t = dr_t^O + dr_t^U \]
\[ r^O \text{ observed, } r^U \text{ unobserved/unrelated} \]

\[ dr_t^U = \sigma_Z dZ_t \]

\[ dr_t^O = a_t dt + \sigma_W dW_t, \]

\[ a_t = E_t[\alpha] \text{ is updated estimate of constant true } \alpha \]
Model of optimal disposition

\[ dr_t = dr_t^O + dr_t^U \]
\[ r^O \text{ observed, } r^U \text{ unobserved/unrelated} \]

\[ dr_t^U = \sigma_Z dZ_t \]
\[ dr_t^O = a_t dt + \sigma_W dW_t, \]

\[ a_t = E_t[\alpha] \text{ is updated estimate of constant true } \alpha \]

\[ da_t = \frac{\sigma_W}{\kappa + t} dW_t, \quad \kappa = \frac{\sigma_W^2}{\eta^2} \text{ from (Liptser and Shiryaev, 1978),} \]

where \( E_0[\alpha] = \alpha_0, \text{VAR}_0[\alpha] = \eta^2. \) Sell if property value falls below \( \alpha_L, \) set \( \hat{\alpha} \equiv \alpha_0 - \alpha_L. \)
Proposition

At the first passage time, \( \tau = \inf_t \{ a_t \leq \alpha_L \} \),

\[
    r_\tau = \sigma_Z Z_\tau - \kappa \hat{\alpha} + \alpha_L \tau.
\]

Thus \( E[r_\tau] = -\kappa \hat{\alpha} + \alpha_L \tau \) and \( \text{VAR}[r_\tau] = \sigma_Z^2 \tau \).
Result

Proposition

At the first passage time, \( \tau = \inf_t \{ a_t \leq \alpha_L \} \),

\[
    r_\tau = \sigma_Z Z_\tau - \kappa \hat{\alpha} + \alpha_L \tau.
\]

Thus \( E[r_\tau] = -\kappa \hat{\alpha} + \alpha_L \tau \) and \( \text{VAR}[r_\tau] = \sigma_Z^2 \tau \).

Mean negative. Variance still proportional to \( \tau \), unless

- there is substantial heterogeneity in \( \kappa \hat{\alpha} \)
Proposition

At the first passage time, \( \tau = \inf_t \{ a_t \leq \alpha_L \} \),

\[
\begin{align*}
\tau_r &= \sigma_Z Z_\tau - \kappa \hat{\alpha} + \alpha_L \tau. \\
\text{Thus } E[\tau_r] &= -\kappa \hat{\alpha} + \alpha_L \tau \text{ and } \text{VAR}[\tau_r] = \sigma_Z^2 \tau.
\end{align*}
\]

Mean negative. Variance still proportional to \( \tau \), unless

- there is substantial heterogeneity in \( \kappa \hat{\alpha} \)
- which should also show up in appraised idiosyncratic returns prior to sale
Appraised holding period returns two quarters prior to sale

\[ S \text{quares} = (\text{quarterly appraisal-based time-series volatility}) \times \text{holding horizon}. \]
Disposition takes place at upper and lower thresholds: $\alpha_L$ and $\alpha_H$
Another alternative

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$$\text{VAR}[r_\tau] = \sigma^2 Z\tau + p(\tau)(1 - p(\tau))(\tau + \kappa)^2(\alpha_H - \alpha_L)^2,$$

$$E[r_\tau] = -\kappa \alpha_0 + (\kappa + \tau) (p(\tau)\alpha_L + (1 - p(\tau))\alpha_H),$$

- Should still see effect in appraised returns
- Predicts a bimodal distribution of holding period returns
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\]

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- Only one of 43 Hartigan and Hartigan (1985) “Dip” tests reject unimodality.
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\[ \tau \to 0 \text{ drift & vol diverge for CRE assets} \]

- Results are **not** spurious
  - Vintage effects
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  - Vintage effects \( \times \)
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  - Qualitatively reproduce results
  - Can be calibrated to the data
Implications

- Results arise from severe asset illiquidity
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Introduction

An empirical puzzle

Model

Empirical robustness

Conclusion

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