The Termination of Mortgage Contracts through Prepayment and Default in the Commercial Mortgage Markets: A Proportional Hazard Approach with Competing Risks

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Abstract

This paper examines the factors driving the equity-owner's decision to terminate lending relationships with debt-holders through either prepaying or defaulting on their underlying debt obligations. Using loan level data, we estimate prepayment and default functions in a competing risks proportional hazard model which allows us to account for unobserved heterogeneity. Under a strict definition of mortgage default we do not find evidence to support the existence of unobserved heterogeneity. However, when the definition of mortgage default is relaxed, we do find weak evidence of two distinctive borrower groups. Our results suggest that the values of implicit put and call options drive default and prepayment actions in a non-linear and interactive fashion. Prepayment and default risks are found to be convex in the intrinsic value of call and put options, respectively. Consistent with the jointness of the two underlying options, high value of the put / call option is found to significantly reduce the call / put risk since the borrower forfeits both options by exercising one. Variables that proxy for cash flow and credit conditions, and ex post bargaining powers are found to further shift the exercise boundary of the associated option, and have significant influence upon borrowers' mortgage termination decision.

KEYWORDS: Commercial mortgage, mortgage default, prepayment, competing risks, transaction cost

Section 1. Introduction and Background

A better understanding of commercial mortgage termination through default or prepayment has important academic as well as practical implications. With their relatively simple financial structure — one underlying property and one collateralized debt obligation — commercial mortgages provide an ideal economic setting to test the rationality of investors and the empirical applicability of contingent claim models. From a practitioner's perspective, the identification of factors relating to default and/or prepayment help efficiently determine not only the appropriate spreads in the underwriting of whole loans, but also diversification strategies affecting pools of loans by such categories as property type and geographic location. For fixed income investors, an appropriately specified empirical termination model can provide a structured methodology to incorporate contemporaneous information in the valuation of not only whole commercial loans, but also their securitized counterparts. Moreover, such a model provides a basis for regulators to efficiently set standards in risk-based minimum capital requirements for both life insurance companies as well as commercial banks.

Despite the importance of the topic, there has been a dearth of empirical research on commercial mortgage termination, primarily due to the lack of data with which to examine the asset class. Kau, et al. [1990] provided a theoretical analysis of commercial mortgage valuation. On the empirical side, most related studies have been conducted using aggregate levels of data (Titman and Torous [1989], and Elmer and Heidorfer [1997]). Yet disaggregate loan histories are needed in order to fully understand the relationship between loan characteristics and the economics of commercial mortgage termination.

Limited studies using loan level data have focused on one termination event; either default or prepayment. Vandell et al. [1993] study foreclosure experience of commercial mortgage loans and find that the equity position, as measured by the contemporaneous market loan-to-value ratio (LTV), is highly significant in explaining mortgage default. However, short-term cash flow conditions, as proxied by original debt service coverage ratio (DCR) are statistically insignificant in explaining default risk. Property type is also found to affect default hazard rates. While this study enhances our understanding of the default experience, several issues hamper the interpretation of the results. First, default is construed as of the date of loan foreclosure. Yet Brown, Ciochetti and Riddiough [2000] find that there is routinely a lag of between six and twenty two months between the start and completion of the foreclosure process. Use of completed foreclosure as the default event date does not accurately measure the economic environment facing the borrower when making a default decision. Second, the use of regional level property indices (without separating the property types) to update property values. The measurement error in the aggregated indices of mixed property types may introduce noise, and the significance of property type dummies reported by Vandell et al. [1993] may simply reflect the measurement error in *property value*, as opposed to the differential propensity of default on loans secured by different types of property. Third, use of DCR *at origination* does not capture contemporaneous cash flow conditions, thus likely explaining its lack of significance in affecting the default decision. Last, the study terminates in 1989, prior to the onset of the real estate recession of the early 1990s, thus failing to capture the significant increase in credit related mortgage default activity over this period.

In a recent study of multifamily mortgage default experience, Archer et al. [2000] identify the significance of original DCR, but not original LTV. Using a logit approach to modeling mortgage default, the authors argue that LTV, DCR, and mortgage rate spread are endogenously and simultaneously determined during initial negotiations between equity and debt-holders. The resulting multi-colinearity makes it difficult to identify a significant correlation between these underwriting variables and the likelihood of default. One explanation for this insignificance may be the lack of efficiency of the logit/probit approach to handling panel data or the noisy measurement of the information sets facing decision-makers. Using cross-sectional data, a logit model is specified and estimated to determine the impact of original LTV and DCR in predicting the incidence of future mortgage default. Yet, underlying economic and financial conditions may have changed dramatically from loan origination to the default event.

Both studies fail to consider the impact of prepayment on the option to default, yet default and prepayment are competing risks: the borrower forfeits one option through the exercise of the other. Empirical work on commercial mortgages generally dismisses prepayment as a result of contracting issues. Many loans include some form of lockout, prepayment penalty, or yield maintenance provision. Yet these forms of contracting did not become widespread in the commercial mortgage markets until the mid 1980s. Even with penalties however, prepayment is found to occur frequently, resulting in pricing fluctuations larger than those associated with default risk (see Fu, LaCour-Little and Vandell [2000]).

Follain, Ondrich and Sinha [1997] estimate a prepayment function with a non-parametric baseline function and gamma-distributed heterogeneous errors using a sample of Federal Home Loan Mortgage Corporation (FHLMC) multifamily loans. The authors find that prepayment is sensitive to the value of the call option, but the responsiveness is short of what one expects in the context of a pure option based pricing model. The study also identifies the importance of unobserved heterogeneity in explaining multifamily prepayment experience, providing corroboration of prior research on residential loans (see Stanton [1995]).

Fu, LaCour-Little and Vandell (2000) examine the effectiveness of various prepayment penalty structures embedded in multifamily commercial mortgage contracts. The authors hypothesize that prepayment occurs either because "(1) the assumed prepayment penalties do not exist, (2) they are less severe than assumed", or "(3) borrowers over-exercise prepayment irrationally from an option-theoretic perspective, or incorporate factors beyond those able to be incorporated in a generalized option theoretic model of prepayment." A prepayment hazard model is specified and estimated using a sample of multifamily loans. The authors find that the nature and terms of the prepayment penalty significantly affect the pattern of prepayment. Findings of the study are consistent with both theoretical and numerical predictions.

A shortcoming of all the studies reviewed is the failure to model default and prepayment events simultaneously and interactively in a competing risk framework. Moreover, these studies do not consider the effects of *contemporaneous* cash flow conditions on put and call risks. In a recent study of the behavior of single-family borrowers, Deng, Quigley and Van Order [2000] model default and prepayment as dependent competing risks in order to effectively examine the jointness of the put and call options. Strong support is found to suggest that the value of the put/call has a significant effect on the call/put risk. The discrete specification of unobserved heterogeneity allows borrowers to be differentiated into groups based on relative riskiness. In terms of prepayment, the high-risk group is found to be approximately three times riskier than the intermediate group, and twenty times riskier than the low risk group. For default, however, borrowers are found to be rather homogeneous.[2] The authors attribute the significance of heterogeneity to differences in borrowers' sophistication in exercising mortgage options (or

[2] Defaults on residential mortgages are rare events because of incomplete separation of investment and consumption decisions in housing as well as the high costs of default on personal credit.

differences in levels of *unobserved* transaction costs). However, unobserved heterogeneity may also capture the measurement errors in option values and observable transaction costs.[3]

In an earlier study, Deng, Quigley and Van Order [1996] analyze a sample of low down payment residential mortgages that default in a competing risk framework, with a model that considers default and prepayment options as interdependent competing risks. While these two studies are the first to examine prepayment and default in a competing risk framework, their analysis is conducted on residential mortgage contracts. Commercial mortgages are very different from their residential counterparts, in that they are typically used to finance investment properties, with debt payments being made from cash flows provided by underlying lease contracts. Thus, the factors driving the mortgage termination decision and the homogeneity/sophistication of commercial borrowers may be very different than in the residential mortgage markets.

In this study we investigate a portfolio of 2,090 commercial mortgages originated by a major life insurance company over the period 1974 through 1990, and tracked through year-end 1995. We examine the following issues associated with commercial mortgage default and prepayment:

1. To what extent do put and call options explain the default and prepayment decisions of commercial mortgage borrowers?
2. How important are modeling default and prepayment risks simultaneously as dependent competing risks?
3. How essential are transaction costs and unobservable heterogeneity among commercial mortgage borrowers in affecting the termination of mortgage debt?
4. To what degree does the definition of mortgage default impact observed behavior by mortgage borrowers?

Our findings suggest:

1. Put and call options are highly significant in explaining commercial mortgage default and prepayment. Ceteris paribus, the more in-the-money the put (call) option is, the more likely

[3] The put option is proxied by probability of negative equity, based on estimated stochastic processes of interest rates and property values. However, to a particular borrower the probability of default is either one or zero, which implies the proxy is either underestimated (high risk group) or overestimated (low risk group). The original LTV, state level unemployment rates and divorce rates may also be noisy measures of contemporaneous transaction costs at the individual borrower level.

the mortgagor will default (prepay). Moreover, the effect of the intrinsic value of the put and call options on the default and prepayment hazard is nonlinear and convex, a finding consistent with option pricing theory.

2. Borrowers forfeit all options by exercising either. Consistent with the jointness of the options, we find that high values of put options increase the value of delay in the exercise of the call option, hence reducing prepayment risk, and vice versa.[4]
3. Transaction costs are important supplements to the option variables in explaining mortgage termination. Specifically, there are significant cash flow, credit and size effects. Solvency conditions reduce default risk, but increase the likelihood of prepayment, while low equity levels significantly reduce the possibility of prepayment. Small borrowers default much less frequently, but prepay more often, relative to their larger counterparts.
4. Under a strict definition of mortgage default, we do not find evidence of heterogeneity among borrowers. However, when the definition of mortgage default is relaxed, we do find weak evidence of heterogeneity in borrower behavior.

The remainder of the paper is organized as follows. The next section describes the characteristics of commercial mortgage markets and derives optimal exercise conditions for mortgage options in the presence of cash flow, credit constraints and contracting costs. Section 3 introduces a proportional hazard model with competing risks and unobserved heterogeneity. In Section 4 we describe the data and summary statistics. In Section 5 we present results of our empirical estimations. Discussion about the robustness of our findings follows in Section 6. Implications and concluding remarks are provided in Section 7.

Section 2. The theory of Commercial Mortgage Option and Its Empirical Implications

In this section, we first describe the features of commercial mortgage contracts, followed by derivation of sufficient conditions for borrowers to exercise mortgage options in a frictionless world. We then discuss the effects of transaction costs on prepayment and default, which include cash flow or credit constraints and incomplete contracting.

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[4] An alternative explanation of LTV on prepayment probability is based on the institutional constraint of refinancing with low levels of equity. A high LTV, corresponds to a small or negative equity position, typically disqualifying the owner of the property from refinancing the loan, even at favorable market rates.

2.1 Characteristics of commercial mortgages

Commercial mortgage markets differ from their residential counterparts in several significant respects. Commercial loans finance investment opportunities, and are typically used by sophisticated investors and real estate developers. Thus, borrowers of commercial debt have very low "psychological" attachment to the underlying asset and should, in theory, be more "ruthless" in the exercise of either the default or prepayment option. Loans are typically fixed rate and fixed payment notes without recourse and are either interest-only or amortizing, with a balloon payment prior to the full amortization term. Prepayment is very often discouraged, but not entirely prohibited, through prepayment penalties and/or yield maintenance features.

Embedded in each mortgage is a termination option that can be exercised by the borrower through either default or prepayment. If the borrower chooses to forego scheduled payments for (up to) 90 days, a foreclosure process typically ensues. Two outcomes are possible. The lender can choose to foreclose and directly own the property, or alternatively, renegotiate the debt contract, often deferring or accepting less than full payment. The borrower can also end the contracting relationship with the lender by prepaying the outstanding loan balance, subject to any applicable prepayment penalties.[5]

2.2 Optimal decision rules without transaction costs

In the context of corporate finance and contingent claim terms, borrowers may be viewed as *equity-holders* and lenders as *debt-holders*. It is well known that the equity-holder faces an option-like payoff. With limited responsibility, the equity-holder can default on the debt and return the asset to the debt-holder. The possibility of prepaying the debt gives the mortgage borrower another valuable financial advantage. First, let us define "termination option" as the value of the opportunity for the equity-holder to terminate the debt contract with the debt-holder through default or prepayment. Under prepayment, the borrower "repurchases" the remaining

[5] After the mass prepayment wave in the late 70s and early 80s, banks and life insurance companies began to implement various protective covenants in mortgage contracts in order to stabilize expected cash flows through a reduction in prepayment incentives. These included lockout periods, prepayment penalties, and yield maintenance provisions. The most severe of these is the yield maintenance provision, under which the borrower is required to pay the full difference between the accounting mortgage balance and the market value of the mortgage. This, at least in theory, would fully eliminate any prepayment incentives from borrower's perspective.

mortgage obligation at the current loan balance, along with any applicable prepayment penalties. Under a default scenario, the borrower "sells" the property to the lender at a price equal to the market value of mortgage, thus reflecting the opportunity cost of the future scheduled mortgage payments to the borrower. At any time, the choice to exercise a termination option will be done through the vehicle (put or call) with the largest intrinsic value, causing *both* future choices to be lost immediately. The borrower can, however, choose to keep the option alive by paying the current period scheduled payment. Thus, in the absence of transaction costs, the borrower will exercise the option at time t if the following condition is satisfied *before* he submits the periodic principal and interest payment:

$$\max\{L_t - V_t; L_t - B_t - f_t; 0\} \geq E_t\left[\frac{P_t + 1}{1 + r_f}\right] - D_t, \tag{1}$$

where V_t is the property value at time t and L_t the market value of the debt. L_t can be expressed as the sum of remaining mortgage payments discounted at the market mortgage rate at time t, M_t.[6] B_t is the accounting outstanding balance of the debt, which equals to the sum of remaining mortgage payments discounted at the contract rate, R_c. f_t is the prepayment penalty as specified in the contracts.[7] Thus (L_t - V_t) defines the intrinsic value of the put option while (L_t - $B_t - f_t$) defines the intrinsic call value to the borrower at time t. The left-hand-side (LHS) of equation (1) defines the payoff of the termination option if exercised at time t. On the right-hand-side (RHS), P_t is the value of the termination option that is (at least) a function of V_t, L_t, B_t, M_t and R_c. D_t is the scheduled payment at time t.[8] r_f is the risk-free rate between t and $t+1$, and $E_t\ [P_{t+1}/(1+r_f)]$ defines the expected (discounted) value of the option in the subsequent period if not exercised today. Expectation on P_{t+1} is taken over possible realization of property value V_{t+1} and mortgage rate M_{t+1}, which relate to V_t and L_t, respectively, through property value and mortgage rate processes. Under conventional assumptions about property value and mortgage rate processes, P_t will be convex in its intrinsic value. Also as the option approaches expiration date — in the case of a balloon mortgage, rollover, or extension date — its exercise boundary will be closer or tighter to the option's strike price. In sum, equation (1) suggests that borrowers should exercise the termination option if it is sufficiently in-the-money — the intrinsic option value today plus the saved cash payment is greater or equal to the discounted option value in the next period —

[6] Where M_t is term-structure matched to include both the risk-free interest rate and a mortgage credit spread component.

[7] f_t can be set as infinity in the case of lockout period.

[8] For step and graduated payment loans, D_t is time-dependent.

otherwise it is optimal to tender the scheduled payment and keep the option alive for at least one more period.

2.3 Optimal decision rules with transaction costs

Although contingent claim theory calls for a sharp exercise boundary, empirical evidence seems to contradict this theory. Rather than appealing to investor's irrationality, researchers have recognized that unobserved, heterogeneous transaction costs may offer a valid explanation to the blurred exercise boundary. Such costs may arise from liquidity or credit constraints, the incompleteness of contracts, and/or borrower risk aversion, all of which vary from borrower to borrower.

2.3a Cash flow and credit effect

In the absence of transaction costs, it is not optimal for a cash-flow-constrained borrower to default with positive equity in the property since additional equity financing could be secured in order to alleviate cash flow constraints. But in more realistic settings, additional borrowing can be quite costly, especially for borrowers with liquidity constraints. Borrowers must bear the transaction costs since debt-holders do not usually have as much information as equity-holders and will require evidence of adequate levels of LTV and DCR. Even with transaction costs, a low DCR will most likely disqualify borrowers from typical market rate financing. An alternative solution for cash-constrained borrowers is to sell the property and pay off the debt at its face value. However, selling per se can also be costly for borrowers in financial distress, due to de facto prepayment covenants.[9]

Default can also be associated with loss of credit rating, or greater difficulty or higher costs of financing subsequent projects. This cost can be especially large for new and/or small investors in the commercial real estate business. While more established borrowers/developers can convince the *next* lender that general market conditions lead to the previous default, an inexperienced investor in the business is more likely to be accused of poor management of the property and held responsible for its default. In addition, smaller investors might sell their property and prepay

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[9] Pulvino (1998) shows that financially constrained airlines receive lower prices than their unconstrained rivals when selling used narrow-body aircraft.

their loans out of consumption-related reasons since they are more likely to be liquidity constrained.

Credit and liquidity levels also impact exercise of the call option by affecting available refinancing rates. A borrower with good credit can qualify for better refinancing terms than others in a falling interest rate environment, which further raises the opportunity cost of the current mortgage and hence, the attractiveness of the prepayment option. On the contrary, institutional requirements for reasonable LTV and DCR levels can also disqualify many borrowers from favorable mortgage rates. Facing higher borrowing costs of personal consumption or business expansion, a small investor may sell the property or prepay a mortgage even in a *rising* interest rate environment in order to "cash out" equity.

2.3b Contract incompleteness

Borrowers with very negative equity positions are frequently observed *not* to exercise the deeply in-the-money put option, as long as short-term net-operating-income (NOI) can cover scheduled debt payments. Instead of handing over the property to the debt-holder, they try to extract as much value as possible from the property, typically through under-investment. Since the property will most likely go to the lender at maturity, further sabotaging of the condition of the property can only make the borrower's equity position more negative. Therefore, the option-like payoff to the equity-holder makes it optimal for the borrower to delay default at the expense of the lender.[10] The balloon structure of most commercial mortgage debt further exacerbates the problem by allowing the borrowers to wait until the balloon date to default, at which point the put option is exercised at its greatest value.

The "waiting to default" scenario described above is sub-optimal from a social welfare perspective as well, since under-investment can damage the property to the extent that it costs the debt-holder much more to repair after taking over the property, thus creating positive deadweight loss (see Jensen and Meckling [1976]). The debt-holder will charge a premium, ex ante, to account for the "stealing behavior" by certain borrowers, leading to only a second-best contract. If contracting is costless and complete, the debt-holder can correct the sub-optimal behavior of the borrower by imposing provisions in the debt contract, that specifically mandate borrowers to

[10] This is similar to the under-investment problem for financially distressed firms with debt "overhang" as discussed in corporate finance (see Myers [1977]).

meet required maintenance levels. However, ex post, these contracts are costly to write, and difficult to monitor and enforce.[11]

In a frictionless world (i.e. without transaction cost), a rational borrower who has both negative equity (LTV > 1.0) and low cash flow (DCR < 1.0) will certaintly become delinquent on debt payments to maximize his financial welfare. However, in real world, there is no costless delinquency or bankruptcy, neither to the borrower, nor to the lender. Therefore a straight bankruptcy decision (foreclosure) is often Pareto-dominated by ex post renegotiation and workout. The higher the transaction cost involved in delinquency or bankruptcy, the more incentive for both lender and borrower to negotiate a workout solution. Further more, debt-holders are usually not as knowledgeable about the value of the property as the equity-holder and not as skillful as the borrowers at management of the property. Thus, they may be willing to restructure the debt and reduce the loan balance, rather then foreclose the loan at the first sign of financial distress. A borrower with (or who believes has) more ex post bargaining power will have higher incentive to default ex ante if he can convince the debt-holder that financial distress is caused by macroeconomic market conditions, rather than inappropriate management. This may result in an agreement to modify and restructure the debt contract. A smaller investor without established history, however, will have more trouble conveying the same argument, often resulting in an immediate full foreclosure.[12] We should note that ex ante the debt-holder would charge a higher coupon rate to account for the ex post renegotiation, unless he can commit himself to foreclosure to deter defaults ex ante.

Empirically incomplete contracting is also observed for prepayment penalties. Borrowers have been observed to default "strategically" in order to prepay without being subject to full yield maintenance penalties, intentionally missing periodic payments to induce default even when property value and incoming cash flow is high relative to scheduled debt service payments. Under a strategic default scenario, borrowers sell the property and repay the loan to realize property value appreciation or a favorable interest rate environment in which to refinance.

[11] The problem posed here shares the familiar moral hazard problem associated with the insurance market, where the owner of the property becomes careless once the insurance coverage is contracted and the premium paid.

[12] This argument is consistent with Brown, Ciochetti and Riddiough (2000), which finds that among loans in delinquency, large ones are more likely to be restructured while small ones are more likely to be foreclosed. They also find that large loans take the lender longer to dispose of because of larger liquidity pressure, which potentially explains the reluctance of lenders to foreclose on large loans.

Lenders, fearing the lengthy legal process and losses associated with the foreclosure process, may accept less than the full difference in yields. The exact penalty (as a percentage of the property value) under this scenario is related to the lender's ability to identify the value of the property precisely as well as the relative bargaining power of the two parties engaged in the strategic default game (see Riddiough and Wyatt [1994]).

Section 3. A Proportional Hazard Model with Competing Risks and Heterogeneous Error

The application of a proportional hazard approach to modeling commercial mortgages is appropriate because of its efficiency in modeling the *entire path* to mortgage termination events. Recent applications include Vandell et al. [1993], Follain, Ondrich and Sinha [1997], Deng, Quigley and Van Order [1996], and Pavlov [2001]. The model we estimate in this study is based on the econometric specification as used in Deng, Quigley and Van Order [2000].

3.1 Proportional hazard model for single risk

Assuming the probability density function of duration of the loan to first default (prepayment) at t is $f(t)$, and the cumulative probability distribution is $F(t)$, the hazard function is defined as the probability *density* of default (prepayment) at time $t+\Delta$, conditional on its being active up to time t:

$$H(t) = \lim_{\Delta \to 0} \frac{\Pr(t < T < t+\Delta \mid T \geq t)}{\Delta} = \frac{f(t)}{1-F(t)}. \tag{2}$$

Following the proportional hazard assumption of Cox [1972], we assume a vector of covariates (or regressors), $x_{i,t}$, either time-invariant or time-varying, that change the baseline hazard function, $H_0(t)$, proportionally in exponential form. Thus the hazard function for subject i at time t can be specified as:

$$H_{i,t}(x_{i,t};\beta) = H_0(t_i)\exp(x_{i,t}'\beta), \tag{3}$$

where β is the vector of constant coefficients. The convenient exponential specification ensures that the hazard rate under different value of covariates is always positive. Theoretically, the hazard function is continuous and can take any non-negative functional form. This flexibility, however, makes the empirical identification and estimation of the model a non-trivial exercise.

Several approaches to estimation have been developed in the literature. The simplest parametric specification assumes a given functional form, (typically Exponential, Logistic, or Weibull) and estimates the one or two unknown functional parameters. However, this choice inevitably exerts constraints on the shape of the underlying hazard function, which can result in inconsistencies such as those shown in economic theory.[13] A popular alternative is *Cox's Partial Likelihood* (CPL) specification (Cox [1975], Cox and Oakes [1984]), which only requires the existence of a common stationary baseline hazard function, H_0, for all subjects. The likelihood function under this scenario is decomposed into two separate parts, each containing unknowns in either the baseline hazard function or the partial likelihood of the proportional changes. So β can be identified without parametric restrictions on the baseline function since $H_0(t)$ factors out as a nuisance number. In this sense, the proportion hazard model is semi-parametric: *non-parametric* in the baseline hazard functions and *parametric* in the specifications of proportional change. However, in economic research, the shapes of baseline hazard are often of great interest themselves, and Cox's specification poses an inconvenience in those cases. Suggested remedies include a two-step procedure.[14] First identify regression coefficients, β, through Cox Partial Likelihood specification; then employ these estimates in the full likelihood function in order to maximize the likelihood function and obtain the necessary parameters for a flexibly specified baseline hazard function, which maybe a high-order polynomial function of time.

A full parametric likelihood function with continuously changing baseline hazard rates is computationally difficult to converge. A tractable solution is to specify a fully parametric likelihood function with a *discrete* flexible baseline hazard function and estimate the parameters of proportional hazard effect and baseline hazard functions simultaneously (see Han and Hausman [1990], Sueyoshi [1992], and Deng, Quigley and Van Order [2000] for examples).[15]

3.2 Discrete proportional hazard model with competing risks and unobserved heterogeneity

Modeling both default and prepayment of commercial mortgage debt as competing risks is a natural choice, as borrowers forfeit both the future option to default and the future option to

[13] An example in labor economics are the spikes in reemployment hazard at 26-27 weeks and 52-53 weeks, which correspond to the termination of unemployment benefits (see Kiefer [1988]).

[14] See Fu, LaCour-Little and Vandell (2000) for an application of this approach.

[15] Discrete in the sense that the hazard is a step function that takes constant values between time t and t+1.

prepay by exercising either. The discrete competing risk proportional hazard function for observation i at time t can be defined as:

$$H_{i,t}^{d}(x_{i,t};\beta_d)=\exp(\gamma^{d}(t)+x_{i,t}^{'}\beta_d) \tag{4}$$

$$H_{i,t}^{p}(x_{i,t};\beta_p)=\exp(\gamma^{p}(t)+x_{i,t}^{'}\beta_p) \tag{5}$$

for default risk and prepayment risk, respectively, where $\gamma^{k}(t)$ is the (log of) integrated baseline hazard rate for risk type k between t-1 to t.

Let t_d and t_p be the duration of a mortgage until it is terminated by default or prepayment, respectively. The joint survival function can then be defined as:[16]

$$S(t_d,t_p\mid X,\theta_d,\theta_p)=\exp(-\theta_d\sum_{t=1}^{t_d}\exp\left(\gamma^{d}(t)+x_t^{'}\beta_d\right)-\theta_p\sum_{t=1}^{t_p}\exp\left(\gamma^{p}(t)+x_t^{'}\beta_p\right)) \tag{6}$$

where $(\theta_d,\ \theta_p)$ are unobservable heterogeneity (location) parameters, which can capture differences in (unobserved) transaction cost structures among borrowers after controlling for observable heterogeneity. For example, some mortgagors may be more financially sophisticated and sensitive to refinancing and default risks; or have an unusually good or bad credit history. In a general specification, $(\theta_d,\ \theta_p)$ are J pairs of distinct, but unobserved types of individuals in the population, each occurring with relative frequency p_j, $j = 1,2,\cdots, J$. However, we limit ourselves to two groups in this study.

The competing risk nature of the dilemma makes only the first realized termination (default, prepayment, or censoring) as the event being observed, i.e. $t = min\ (t_d,\ t_p,\ t_c)$. Define $F_d(k|\theta_d,\theta_p)$ as the probability of mortgage termination by default in period k, $F_p(k|\theta_d,\theta_p)$ as the probability of mortgage termination by prepayment in period k, and $F_c(k|\theta_d,\theta_p)$ as the probability that the mortgage will survive until period k, and subsequently censored due to missing data.[17] Follwong McCall [1996], we can write the probabilities as: [18]

$$\begin{aligned}F_d\left(k\mid\theta_d,\theta_p\right)=S\left(k,k\mid\theta_d,\theta_p\right)-S\left(k+1,k\mid\theta_d,\theta_p\right)-0.5\{S\left(k,k\mid\theta_d,\theta_p\right)\\ +S\left(k+1,k+1\mid\theta_d,\theta_p\right)-S\left(k,k+1\mid\theta_d,\theta_p\right)-S\left(k+1,k\mid\theta_d,\theta_p\right)\},\end{aligned} \tag{7}$$

[16] We drop the index i for notation simplicity.

[17] In the estimation of the competing risk hazard model, censored observations include all matured loans and active loans as of the end of the study period.

[18] The term in $0.5\{\}$ is an adjustment for discrete time specification of duration dependence.

$$F_p\left(k \mid \theta_d,\theta_p\right) = S\left(k,k \mid \theta_d,\theta_p\right) - S\left(k,k+1 \mid \theta_d,\theta_p\right) - 0.5\left\{S\left(k,k \mid \theta_d,\theta_p\right) + S\left(k+1,k+1 \mid \theta_d,\theta_p\right) - S\left(k,k+1 \mid \theta_d,\theta_p\right) - S\left(k+1,k \mid \theta_d,\theta_p\right)\right\}, \tag{8}$$

$$F_c\left(k \mid \theta_d,\theta_p\right) = S\left(k,k \mid \theta_d,\theta_p\right). \tag{9}$$

The unconditional probability is then given by:

$$F_m(k) \ = \ \sum_{j=1}^{J} p_j F_m(k \mid \theta_{dj},\theta_{pj}), \qquad m = p,d,c; \tag{10}$$

and the log likelihood function of the fully specified competing risk proportional hazard model with unobserved heterogeneity is given by:

$$\log\left(L\right) \ = \sum_{i=1}^{N}\left\{I_{id}\log\left(F_{id}\left(k_i\right)\right) + I_{ip}\log\left(F_{ip}\left(k_i\right)\right) + I_{ic}\log\left(F_{ic}\left(k_i\right)\right)\right\}, \tag{11}$$

with N being the total number of observations and I_{id}, I_{ip}, and I_{ic} the indicator functions that take values of one if the i^{th} loan is terminated by default, prepayment or censoring respectively, and zero otherwise. j in equation (10) indexes for discrete unobserved heterogeneous groups. Neglecting heterogeneity leads to biased β estimates (toward zero), and may impact interpretation of the results. For instance, a borrower may be sophisticated in the decision to exercise one option, but insensitive with the other — negative correlation — or unsophisticated at both, which implies a positive correlation between the propensity to prepay and default.

Section 4. Empirical Analysis

In this section, we first discuss the source of data used in the study and the construction of empirical variables, followed by a discussion of summary statistics.

4.1 Sources of data and construction of variables

The data used in the study come from several sources. Loan level data are secured from a large, multi-line life insurance company, and consist of 2,589 individual loans originated over the period 1974 through 1990. Relevant loan level characteristics include loan size, contract interest rate, loan term, quarterly status indicator, contractual payment information, borrower type, property type collateralizing the loan, and geographic location.[19] Property value and cash flow indices are secured from the National Council of Real Estate Investment Fiduciaries (NCREIF),

[19] Loans are categorized on a quarterly basis as active, 30 days delinquent, 60 days delinquent, 90 days delinquent, restructured, in process of foreclosure, foreclosed, paid in full, prepaid, or sold to a third party.

and data from the American Council of Life Insurance (ACLI) are used to provide a proxy for prevailing commercial mortgage interest rates.

Empirical analysis requires contemporaneous information regarding property (asset) values, property cash flow levels (net operating income), and mortgage debt values. Since these are not observable directly, we use the quarterly NCREIF property and income return series, stratified by eight geographic regions and four property types to approximate the price paths of individual properties.[20] We then match estimated property values with the NCREIF income return index to construct a contemporaneous net operating income (NOI) series.[21] Using this methodology, we are able to match 2,090 individual loans with the NCREIF series. ACLI commitment rate data are used to provide an estimate of the contemporaneous market value of each mortgage from the borrower's perspective. We do so by fitting a third order polynomial function to the quarterly ACLI commitment series, using the remaining loan term to account for the term structure effect associated with each mortgage. A modeled spread is added to the fitted mortgage rate to account for property type variation. Contemporaneous market loan values are estimated as the sum of the remaining contractual payments, discounted at the appropriate rate as described above.[22]

We measure the extent to which the put option is in-the-money with the contemporaneous "LTV RATIO", computed as the ratio of the market value of the loan to the market value of the property. As discussed in Section 2, since (L_t - V_t) is the exercise value of the put option, we can view "LTV RATIO" as one plus the *scaled* (by property size) intrinsic value of the put.[23] LTV also affects prepayment decisions. From a pure option perspective, a high value (greater likelihood) of prepayment in the future will give borrower more incentive to keep the mortgage

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[20] For years between 1974 and 1978 where NCREIF data are not available, the Marshall and Swift construction cost index is used to supplement the NCREIF property value index.

[21] The index as stratified by region and property type is not complete since it is not available for some years and for some regions. In those cases, a more aggregate index is used. In order for the index to be useful, we impose the condition that it must have at least 36 quarters of data (or 9 years) at the region and property level. 36 quarters are chosen to capture the tremendous property value fluctuation in the early 90s.

[22] Deng (1997) argues for the superiority of using stochastic term structure model to calculate the value of the loan numerically. But that methodology will require the choice of (a) an appropriate term structure model; and (b) a set of parameters for the specified process. The method used in the present study has the advantages of parsimony and computational efficiency.

[23] The constant term, one, is absorbed by the baseline hazard function, and will not affect the estimates of the coefficients.

termination option alive. From an institutional perspective, the LTV RATIO can affect the refinancing decision: a high LTV RATIO (low equity level) will make it more difficult for the borrower to secure alternative financing, thus reducing the prepayment hazard. The LTV RATIO is also a measure of financial leverage. Commercial mortgage borrowers with a low ratio are observed to rationally refinance in order to lever up his/her equity in the property in order to realize tax benefits and/or enhance investment returns. Borrowers may also be expected to prepay or sell properties with a low LTV RATIO in order to "cash out" their equity positions either for personal consumption or business expansion. This may occur even in the absence of contemporaneous market interest rate benefits. Overall, we expect LTV RATIO to have a positive effect on the default hazard and a negative effect on the prepayment hazard.

The financial incentive to prepay is measured by the contemporaneous "CAL RATIO", estimated as the ratio of the outstanding loan balance less the market value of the loan to the market value of the loan. Thus, the CAL RATIO is the *scaled* intrinsic value of prepayment for the borrower. We expect CAL RATIO to reduce the default hazard in the context of a joint mortgage option. The effect, however, may be rather small since refinancing will be very unlikely when a property is close to default. Conversely, the coefficient on CAL RATIO could be positive in a "strategic default" scenario, one in which a borrower intentionally defaults in order to trigger an acceleration of the loan to avoid a prepayment penalty in a favorable interest rate environment.

In order to capture the convexity of the option value relative to its intrinsic value, we also include squared terms for LTV RATIO and CAL RATIO. These terms are denoted as "LTV SQUARED" and "CAL SQUARED".

Short-term solvency status of the equity-holder is measured by contemporaneous "DCR", estimated as the ratio of contemporaneous NOI to scheduled debt service payments. Insufficient cash flow can result in default if additional equity borrowing is unavailable, or becomes too expensive. Low DCR is also likely to diminish the prepayment hazard, as the property does not generate sufficient cash flow to refinance the debt at market rates. We note that NOI is constructed from contemporaneous property value and income return indices, thus not relying on the DCR at loan origination, which may contain more idiosyncratic information about the cash flow conditions of the property at loan origination. DCR at loan origination, however, is included in the hazard regression, denoted as "ORIGDSC".

With the financial incentive associated with option exercise measured in relative form by LTV RATIO and CAL RATIO, a loan size variable is needed to capture the *quantitative* difference of the option value in a net present value sense. If an economy of scale holds for the cost associated with option exercise, then larger loans should be associated with a higher likelihood of exercise for *both* the put *and* call options. We include in our regression loan size dummies, SMALL, MEDIUM, and LARGE.[24] To proxy for management expertise, financial sophistication, borrowing costs, and ex post bargaining power, we also include a set of borrower type dummies, INDIVIDUAL, PARTNERSHIP, CORPORATION, and OTHER.

Loan type dummies, AMZDUMMY for amortized loans, ACRDUMMY for accrual loans, GPMDUMMY for loans with changeable payments or step payments, and FIXED for fixed rate loans are also included. These variables capture the variation in required periodic payment, which is the cost associated with keeping the mortgage termination option alive. Accrual and step-rate loans exhibit increasing periodic payments, hence a greater likelihood of option exercise over time. Loan type dummies also reflect differential debt-equity structures at the balloon payment. Relative to interest-only loans, those with amortization provisions have less principal due at maturity and hence larger equity position. We expect amortizing loans to have a lower probability of default at loan maturity.

Property type dummies, APARTMENT, OFFICE, INDUSTRIAL, or RETAIL are also included to capture property related cash flow and risk characteristics. Property type has been shown to be significant in explaining commercial mortgage default hazard rates in prior studies (see Vandell et al. [1993]). However, with a properly specified model, and under rational borrower decision-making, it is hard to rationalize such significance.

We also include a balloon dummy which takes the value of one if the loan is within one quarter of maturity date, zero otherwise. A risk averse investor with positive equity will be searching for alternative financing prior to the maturity date to avoid unintended de facto default with positive equity. Those with negative equity cannot benefit at the expense of debt-holders at the balloon date, making default the optimal decision to exercise. From an option perspective, as the contract approaches maturity, the time value of the option is very small relative to its intrinsic value, encouraging borrowers to exercise the option and pocket the intrinsic value.

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[24] Loans less than $2 million in size are categorized as small, those greater than $2 million and less than $7 million are categorized as medium, and those greater than $7 million are categorized as large.

4.2 Summary statistics

As shown on Panel A of Table 1, the mean loan to value ratio is slightly greater than 72 percent, and the corresponding debt coverage ratio averages 1.25. The average maturity of loans in the sample is shown to be slightly less than 12 years. Loans are diversified across geographic regions, with greater concentrations in the East North Central and Southeast regions of the country (Table 1, Panel B). Of interest is the incidence of default and prepayment as stratified by region of loan origin. Loans originated in the Southwest region of the country exhibit not only the greatest proportion of defaults, at 59 percent, but also the lowest prepayment rate, at slightly greater than 12 percent. When stratified by property type securing the loan, office properties are shown to dominate the sample, constituting nearly 44 percent of the pool (Table 1, Panel C). There are more cases of default for office properties and more cases of prepayment among industrial properties. In terms of loan origination, a bi-modal distribution is evident, with high loan activity in the late 1970s and again during the mid 1980s (Table 1, Panel D). Loans initiated in the mid 1970s and early 1980s have experienced greater prepayment, while loans originated in the early 1980s have experienced greater levels of defaults. Overall, among the 2,090 loans under examination, slightly less than 29 percent are shown to have prepaid and 27 percent are shown to have defaulted.

Over the period under examination, 1974 through 1995, tremendous fluctuations in the value of commercial real estate property are found to have occurred (Figures 1 and 2), especially during the real estate recession of the late 1980s and early 1990s. The variation in property value assures a rather powerful test of the effect of option theory on mortgage termination. The sample's mean mortgage coupon rate tracks that of the industry quite well, as proxied by ACLI commitment rates, except during a brief period in the early 1980s, when interest rates and volatility were very high (Figure 3). Mean coupon rates for the sample were approximately 100 basis points lower during this period than that as reported by ACLI (Panel D of Table 1, and Figure 3).

Loans that eventually terminate through default are shown to have a slightly higher initial LTV and slightly lower initial DCR (Table 2). The value of the call option around origination is higher for loans that eventually default, suggesting that these loans may have been assessed a higher coupon rate premia up front to negate the risk. At the time of termination, default / prepaid loans have much lower / higher DCR ratios than active or matured loans and a much higher / lower LTV ratios.

Section 5. Hazard Regressions with and without Unobserved Heterogeneity

We first estimate the competing risk hazard model *without* heterogeneity (Model 1). The baseline function is modeled as a 5-th order polynomial function of time in quarters and the hazard rates are assumed to be constant *within* each quarter. Our strict definition of default is defined as the first time a loan falls into the category of "In Process of Foreclosure" or "Foreclosed", while prepayment is defined as the quarter at which the loan balance is paid in full prior to scheduled loan maturity.[25] From an estimation perspective, Model 1 is equivalent to an *external* competing risk hazard model, which can be estimated through two, single risk hazard models, each considering termination with competing events as censoring. Subsequently, we consider the effect of unobserved heterogeneity (Model 2).

5.1 Results without unobserved heterogeneity

Table 3, Panel A presents the results from fitting Model 1. If we combine the effects of linear and square terms for the option variables, borrowers with large intrinsic values of the put (LTV RATIO) or call option (CAL RATIO) are more likely to default or prepay (see Figures 4A and 4B). Furthermore, the effects of the intrinsic value of options on instantaneous prepayment and default hazards are convex. Prepayment is not sensitive to the intrinsic call value when it is out-of-the-money, yet starts to increase very rapidly after CAL RATIO becomes positive and until it hits about 30 percent (see Figure 4B). Mortgage default, however, begins to increase prior to the point of negative equity (Figure 4A). This may in part reflect noise in our measurement of property value as compared to our measurement of the call value. We also find call (put) risks strongly affect the exercise of put (call) option. Very large values of the put option (high LTV RATIO) reduce the likelihood of prepayment, while high values of the call option (high positive CAL RATIO) moderate the risk of default. These indirect effects confirm the significance of the *jointness* of the two mortgage options and the importance of modeling them as competing risks — by exercising the call (put) option, the borrower forfeits both the future default and prepayment options. The effect of contemporaneous LTV on prepayment risk may be explained

[25] For various definitions of default, and possible paths of a loan's history, see Archer et al. (2000).

by institutional constraints on required equity levels necessary for borrowing. Highly negative CAL RATIO (deeply out-of-the-money), however, is found to reduce the default hazard risk (see Figure 4B). We believe this result captures the large number of low interest loans made in mid to late 1970s that were subsequently paid off by borrowers per the terms of the mortgage contract in the high interest rate environment of the early 1980s, prior to the general implementation of sophisticated prepayment penalties.

We find that *contemporaneous* insolvency, proxied by a low DCR, significantly raises default risk while reducing prepayment risk, even *after* controlling for the value of the put and call options. The significance of cash flow variables on default suggests that borrowers with negative equity do not default as long as income generated by the property is sufficient to cover scheduled debt payments. An alternative explanation is that borrowers in cash flow distress might default with positive equity in their property.[26] This seems to imply that the selling costs are quite high or additional short-term equity financing are very costly, rendering them more expensive alternatives to default. Contrary to prior research, DCR at *origination* shows up negative in the hazard function for prepayment, and insignificant for default hazard. Borrowers may engage in "window dressing" of their cash flow projections, similar to the behavior identified for corporations prior to raising capitals through debt or equity. Thus, short-term earnings projections may not be sustainable, eventually mean reverting to the index average. Moreover, this finding emphasizes the importance of estimating *contemporaneous* variables when specifying models of mortgage prepayment and default.

Borrowers of large loans are found to be more likely to default but, less likely to prepay, while borrowers of smaller loans are more likely to prepay. Loan size can capture differences in costs of capital and bargaining power in workout situations, which shifts the optimal exercise boundary of the options. Borrowers of smaller loans are usually charged higher coupon rates initially. Yet, as these borrowers gain more expertise in property management and accumulate more experience, they can obtain better financing arrangements. Lacking alternative means for low cost borrowing, borrowers of small loans might also resort to refinancing or selling in order to cash out equity for personal consumption and/or business expansion. Borrowers of large loans may have fewer incentives to protect their credit from default, possibly because of their well-established credit history, experience in property management, or ownership structure. They are

[26] This phenomenon has been observed in earlier studies of commercial mortgage default (see Ciochetti and Riddiough [1998]).

also more likely to exert influence in the ex post negotiation with the lender since they can best manage the underlying property securing the mortgage. Borrower type does not seem to affect either default or prepayment risks. Loan size dummies appear to better capture the variation in borrowers' bargaining power and credit availability than do borrower type dummies.

Property type does not seem to affect default or prepayment risk. The insignificance of property type on default is in contrast to the findings of Vandell et al. [1993], where more aggregate indices are employed to construct property value in order to measure solvency conditions. Our findings suggest that the property variables employed in their study may capture the residual property-type specific disparity in property value and cash flow conditions.

We find accrual and step-rate loans to be positively related to default risk, reflecting the increasing costs associated with keeping the mortgage option alive as time goes by. These variables may also reflect self-selection at loan initiation. Under asymmetric information, borrowers with higher default risks will choose to take loans with lower initial payments. As loan balances increase after origination, borrowers of accrual and step rate loans are much more likely to default and less likely to prepay.

The balloon year dummy exhibits a strong impact on both prepayment and default events. Prepayment immediately before maturity reflects the borrower's risk aversion, but has only a small effect on lender's return. Default at balloon year however, reflects the value of the "wait-to-default" option for the borrower, and the resulting losses to the lender can be severe.[27]

5.2 Estimation with unobserved heterogeneity

Table 3, Panel B reports results with bivariate unobserved heterogeneity (Model 2). The estimation shows no significant heterogeneity among borrowers in the risk of exercising call and put options as reflected by the lack of significance of the MASS2 variable. The first risk group, consisting of 90 percent of the population, dominates the second group statistically, but the

[27] See for example Snyderman [1994], Esaki, et al. [1999], Ciochetti and Riddiough [1998], or Ciochetti and Shilling [1999].

existence of the second group is not statistically significant.[28] This result supports the assumption of rational behavior by mortgage borrowers in the exercise of call and put options on commercial mortgages, and suggests that our model is well specified. This result is in contrast to Deng, Quigley, and Van Order [2000], where significant heterogeneity is found in prepayment but not in default. We postulate that this may result from the low occurrence of default activity relative to prepayment activity in their mortgage data, since 73.6 percent of the loans in their sample are prepaid, while only 1.6 percent default. In the present study, we observe 28.8 percent prepayment and 27 percent default activity, respectively.

Qualitatively the coefficients on the observed characteristics of the loans are similar to the case without heterogeneity. Deng, Quigley and Van Order [2000] and Follain, Ondrich and Sinha [1997] argue that ignoring heterogeneity can lead to biased estimates. Thus, the fact that there is little change in the parameter estimates offers indirect support for the lack of heterogeneity among borrowers with respect to prepayment and default, and to our model specification.

An ongoing concern related to commercial mortgage default analysis is the appropriate definition of mortgage default. ACLI reports loans as delinquent after 60 days, while the National Association of Insurance Commission (NAIC) reports loans as delinquent at 90 days. Prior research on commercial mortgage default has included studies that construe default from as early as 90 days delinquent (Snyderman [1994], and Archer et al. [2000]), to as long as actual loan foreclosure (Vandell et al. [1993]). Yet, borrower behavior with respect to exercise of the put option may vary considerably depending on the precise definition of default.

In order to examine the extent to which default definitions may impact the empirical nature of the competing risk of prepayment and default, as well as the degree to which unobserved heterogeneity may be identified, we re-estimate Models 1 and 2 with two, more expanded definitions of mortgage default. In the first case, we define default as all loans in process of foreclosure and foreclosed as in Table 3, but *add* loans that have experienced some form of renegotiation or modification (Tables 4). We do so, because renegotiation inevitably has an economic impact on both the borrower and lender in some form, through a reduction in the note rate, accrual provisions, forbearance, and the like. We next expand the definition of default further to include not only loans in process of foreclosure, foreclosed loans, and renegotiated

[28] MASS1 is normalized to 1 in the empirical estimation, so that the probability of being in the first group is MASS1/(MASS1+MASS2) = 1/(1+0.1169) = 90%.

loans, but *also* those loans that are 90 days delinquent (Tables 5). The extended definition of default reflects the borrower's perspective in exercising the default option, while the restricted definition of default more closely related to the outcome of default, hence reflect to the lender's perspective. By extending the definition of default, our counts for prepaid and defaulted loans ranges from 610 and 422, respectively in Tables 3, to 599 and 534, respectively in Tables 4, and 576 and 596 respectively, in Tables 5.[29]

Qualitatively, we observe little difference in results from the maximum likelihood estimation without unobserved heterogeneity (Model 1), using the expanded definitions of mortgage default. In most cases, we note that parameter estimates are comparable as we move from a strict definition of default (Table 3, Panel A) to more broadly defined definitions (Tables 4 and 5, Panel A).

Of interest, however are the estimation results *with* bivariate unobserved heterogeneity. While we found no significant heterogeneity among borrowers using a strict definition of default (in process and foreclosed), we do find a statistically significant second mass point when we expand the definition of mortgage default as described above. As shown in Tables 4 and 5, Panel B, the population of group 2 in Table 4 is 40 percent (0.6702/(1+0.6702)=0.4), and 39 percent (0.6445/(1+0.6445)=.39) in Table 5. Though we find in Table 4 that the second group is about 18 times more likely to prepay, and about 10 percent more likely to default than the first group, but the estimated differences are statistically insignificant. [30] In Table 5 we find that the second group is 22 times more likely to prepay, and 7 percent more likely to default than the first group. Again, the estimated differences are statistically insignificant. We postulate that the reason we find weak evidence of heterogeneity (i.e., about 60 percent of the borrowers may belong to one group and 40 percent of the borrowers may belong to a second group) in the more relaxed definition of mortgage default is due to the increased level of idiosyncratic borrower behavior that is unable to be captured by the current specification of the model. For example, much more ex post negotiation may occur with either modified and/or delinquent loans. Thus, the heterogeneity parameter picks up this variation. This is in contrast to the more strict definition of default, where in process and foreclosure behavior is generally non-reversible, and is associated with severe

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[29] Note that as we move from a strict definition of default, the number of prepayments decrease.

[30] The numbers for prepayment and default are derived as LOC2/LOC1.

economic impact; thus, borrowers under this definition may act more judiciously and in a more homogeneous manner.

Consistent with Deng, Quigley and Van Order [2000], Tables 4 and 5 illustrate the importance of estimating a competing risk model with unobserved heterogeneity. Although there is no qualitative change in parameter significance, there is significant change in magnitude, especially for the impact of CAL RATIO and DCR on default. These results suggest one should be concerned about possible estimation bias when heterogeneity is ignored.

5.3 Conditional prepayment and default rate

Figure 5 depicts the fitted conditional prepayment rate (CPR) and conditional default rate (CDR) for a loan representing the sample median values of LTV and DCR, and zero for all other variables, based on estimation results from Model 1. The CPR starts at zero at loan origination, rises and peaks at about 10 percent near the 30th quarter, falling to 5-6 percent by the 70th quarter. For a typical loan, the CDR is much smaller in magnitude than CPR, rising to about 1 percent in year 4, and staying relatively constant over the remaining loan life. Cumulative prepayment and default rates are shown in Figure 6. These results are generally consistent with those found in prior research.

Section 6. Robustness Analysis

In this section we discuss the robustness of our results relative to issues of measurement errors, and specification of the baseline hazard function.

We recognize that the aggregate indices are not perfect substitutes for individual level property value and cash flow information, since they may underestimate loan-level volatility.[31] By definition, half of the properties perform better, and half worse than the indices. Moreover, average ACLI mortgage commitment rates fail to reflect the actual availability and exact rates of refinancing credit to a specific borrower with particular credit history. The heterogeneity in prepayment penalties may exacerbate any noise in the measurement of call values. The

[31] By construction, the NCREIF index is smoothed and contains spurious seasonality. The seasonality is caused by a concentration of outside appraisals at the end of calendar year (see Geltner and Goetzmann [2000]).

measurement errors in the calculated contemporaneous property, loan, and NOI values will be inherent in the LTV, CAL and DCR ratios. Empirical estimation will thus lead to (downward) biased estimates of the coefficients, which may make generalization of results more difficult.

Three features in the research design can mitigate the measurement error problems in the hazard regression. First, variables highly correlated with the measurement errors are included as observed heterogeneity variables. For example, contemporaneous LTV and DCR can affect the availability and cost of credit, and are included in the hazard function of prepayment. Second, unobserved bivariate heterogeneity can partially control for the measurement error through location parameters. For example, loans with underestimated/overestimated property values will likely be grouped into the category with a high/low value of location parameter of default, (θ_d), to compensate for the underestimation in the default hazard rate. Finally, a flexibly specified baseline will capture the underestimation of volatility from using aggregate indices in estimating individual loan information. As time passes, the value and cash flow performance of individual properties will deviate further away from the index level by accumulating more idiosyncratic risks. In other words, the issue of measurement error will become more severe over time. However, the baseline hazard can trend up to offset the under-prediction in the prepayment and default hazard due to the lack of cross-sectional variability in loan level variables (LTV, CAL and DCR) over time.

An additional concern is the effect of a misspecification of the baseline function on our model estimates. The empirical estimation of the hazard model in this paper assumes a 5-th order polynomial function as the baseline function. To check the robustness of our results to the functional form of baseline hazard, we estimate our model using Cox Partial Likelihood (CPL) specification, which does not require specification of a baseline function form. As shown in Table 6, the estimation generates qualitatively similar results, except that the significance of property type dummies on the prepayment hazard disappears. We also consider the effect of regional dummies, generating qualitatively similar results.[32] Some regional dummies do show up as significant, most likely capturing the residual variations in regional level property value and cash flow differences due to our incomplete construction of region by property type indices.

Section 7. Conclusions and Implications

This study is the first to examine commercial mortgage default and prepayment in a competing risk hazard framework using loan level data. We explicitly model prepayment and default as a joint mortgage termination option. Our empirical findings are largely consistent with the predictions from the theory of contingent claims, and prior empirical research using residential mortgage data. High values of put and call options greatly increase the default and prepayment risk in a nonlinear (convex) manner. The value of the put/call option is also found to significantly affect the exercise of the call/put option, thus capturing the competing-risk nature of the two termination events.

We also show that option pricing theory alone is not adequate to explain commercial mortgage defaults and prepayments. The financial sophistication, bargaining power, solvency, and credit history of borrowers also affect the mortgage termination decision by shifting the exercise boundary of both the prepayment and default options.

Relative to research conducted on residential mortgages, we find no evidence of unobserved heterogeneity among mortgage borrowers under a strict definition of mortgage default. However, we do find weak evidence of unobserved heterogeneity under more general definitions of mortgage default. On further examination, heterogeneous behavior of mortgage borrowers comes mainly from exercise of the prepayment option, which is consistent with conclusions from residential studies.

Relative to research conducted on commercial mortgages, this study confirms the importance of using contemporaneous information as proxies for the theoretic put and call variables. Interestingly, after controlling for contemporaneous debt coverage ratio, we find no evidence to suggest that *original* debt coverage ratio is related to commercial mortgage default. This is in contrast to prior work, which fails to include contemporaneous cash flow information in the empirical model specification.

Our results have important practical implications. We establish empirically that aggregate indices contain valuable information about the performance of individual loans and demonstrate how to incorporate such information efficiently through a hazard model framework. Future default and prepayment paths can be predicted by simulating property value and interest rate processes to allow for the pricing of whole loans and their securitized counterpart. The competing risks

[32] Available on request from the authors.

methodology is also applicable to regulators in order to set efficient minimum capital requirement for institutions involved in commercial mortgage lending. As exogenous observable variables shift the option's exercise boundary and affect mortgage terminations through the transaction cost structure, they should be explicitly considered in both the underwriting and the pricing of commercial mortgages. These important issues warrant continued research.

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Table 1. Descriptive Statistics

Panel A: Means and Standard Deviations

	Mean	*STD*
Origination Year	1981.6	4.2
Loan Term (Years)	11.9	5.4
Coupon Rate (%)	10.5	1.6
Original DCR	1.25	0.18
Original LTV (%)	72.2	6.2
Loan Size (MM)	$6.7	$15.3

Note: Mean and standard deviation calculated based on 2,090 loans.

Panel B: Distributions by Region

Region	*Number of Loans*	*Percent*	*Percentage Default*	*Percentage Prepaid*
EN	396	18.9	17.4	35.1
ME	201	9.6	25.9	25.4
NE	222	10.6	37.8	22.5
SE	267	12.8	28.1	23.6
SW	157	7.5	59.2	12.1
WM	222	10.6	39.6	26.1
WN	99	4.7	32.3	33.3
WP	526	25.2	13.5	35.7
Total	2,090	100.0	27.0	28.8

Table 1 (Cont.) Descriptive Statistics

Panel C: Distributions by Property Types

Borrower Type	*Number of Loans*	*Percentage of the Total*	*Percentage Default*	*Percentage Prepaid*
APARTMENT	420	20.1	23.8	28.1
INDUSTRIAL	437	20.9	16.5	38.7
OFFICE	914	43.7	35.6	24.1
RETAIL	319	15.3	21.0	29.5
Total	2,090	100.0	27.0	28.8

Panel D: Distributions by Origination Years

Year	*# of Obs.*	*Percentage of the Sample*	*Mean Coupon Rate*	*Percentage Default*	*Percentage Prepaid*
1974	61	2.9	8.75	4.9	40.1
1975	60	2.9	9.56	13.3	33.3
1976	86	4.1	9.71	16.3	51.2
1977	198	9.5	9.39	7.1	48.5
1978	232	11.1	9.49	9.1	29.7
1979	201	9.6	9.84	14.9	28.9
1980	160	7.7	10.56	17.5	23.1
1981	86	4.1	11.85	12.8	52.3
1982	53	2.5	14.64	13.2	73.6
1983	181	8.7	12.35	25.4	38.1
1984	117	5.6	12.79	47.0	15.4
1985	194	9.3	11.88	56.7	13.4
1986	152	7.3	9.95	52.6	15.8
1987	141	6.7	9.50	52.5	9.9
1988	94	4.5	9.67	45.7	13.8
1989	35	1.7	9.45	37.1	2.9
1990	39	1.9	9.42	18.0	7.7
Total	2,090	100.0	10.5	27.0	28.8

Table 2. Descriptive Statistics at Origination and Termination

Variable	*At Origination*			*At Termination*		
	All Loans (N=2,053)	Prepaid (N=599)	Defaulted (N=534)	All Loans (N=2,053)	Prepaid (N=599)	Defaulted (N=534)
Original LTV	0.721 (0.058)	0.715 (0.057)	0.726 (0.048)	–	–	–
Original DCR	1.259 (0.161)	1.261 (0.118)	1.217 (0.130)	–	–	–
LTV From Index	0.713 (0.075)	0.706 (0.067)	0.735 (0.084)	0.670 (0.354)	0.540 (0.197)	0.848 (0.474)
DCR From Index	1.044 (0.259)	1.047 (0.182)	0.979 (0.199)	1.280 (0.674)	1.602 (0.755)	0.895 (0.390)
CALL OPTION	-0.026 (0.068)	-0.023 (0.068)	-0.010 (0.065)	0.026 (0.047)	0.027 (0.052)	0.0382 (0.0603)

Note: Standard deviations are in parentheses. Of the original descriptive sample of 2,090 loans, 47 lacked complete information and were deleted for purposes of estimation. Thus, the estimation sample is comprised of 2,053 loans.

Table 3 Maximum Likelihood Estimates for Competing Risks Hazard Model of Prepayment and Default with Default defined as in process of foreclosure or foreclosure.

Panel A. *Without* Unobserved Heterogeneity

	Model 1					
Variables	Prepayment			Default		
	Parameter Estimate	T-ratios		Parameter Estimate	T-ratios	
MEDIUM	-0.1825	-1.4250		0.5583	3.3974	***
LARGE	-0.4209	-2.1012	**	1.0324	5.5735	***
AMZDUMMY	-0.0699	-0.2849		-0.8953	-6.9118	***
ACRDUMMY	-0.7871	-1.2239		0.4378	1.8051	*
GPMDUMMY	-0.0384	-0.2070		0.2304	1.6589	*
APARTMENT	0.0036	0.0209		0.2014	0.9851	
INDUSTRIAL	0.1040	0.6133		0.1203	0.6205	
OFFICE	0.0118	0.0667		-0.0711	-0.2999	
INDIVIDUAL	0.1189	0.4975		-0.0334	-0.1264	
PARTNERSHIP	0.2559	1.3476		0.1660	0.9636	
CORPORATION	-0.0165	-0.0754		-0.1372	-0.6510	
ORIGDSC	-0.6634	-1.4769		0.3989	0.8924	
EN	0.5467	2.4086	**	-0.7611	-3.5615	***
ME	0.0949	0.3768		-0.4163	-1.8382	*
SE	0.3679	1.4668		-0.1256	-0.6026	
SW	0.3319	0.9431		0.3569	1.7384	*
WM	0.4115	1.5316		0.2972	1.5153	*
WN	0.7275	2.5132	**	0.1598	0.6769	
WP	0.3268	1.4961		-0.9776	-4.6786	***
LTV RATIO	1.4149	1.4390		-1.0629	-2.0057	**
CAL RATIO	16.6254	5.3170	***	7.9263	3.6403	**
DCR	0.6296	3.9712	***	-0.6340	-2.8004	**
BALLOON	4.0426	21.4466	***	1.7682	11.8704	***
LTV SQUARED	-2.0783	-2.9513	***	0.5699	3.4132	***
CAL SQUARED	-24.3367	-1.8268	*	-45.3062	-3.5526	***
LOC1	2.90E-05	0.8372		2.20E-04	0.9031	
LOC2						
MASS2						
Log Likelihood	-4232.6413					

Note: The model is estimated by maximizing the likelihood function of the competing risk hazard model of prepayment and default. Two 5-th order polynomial functions are estimated as the baseline hazard functions (not reported). *, ** and *** indicate significance at 10%, 5%, and 1% levels respectively. Base variables included in the model are small loans, fixed rate loans, retail properties, and other category for ownership. Number of observations for prepayment and default are 610 and 422, respectively.

Table 3 Maximum Likelihood Estimates for Competing Risks Hazard Model of Prepayment and Default with Default defined as in process of foreclosure or foreclosure.

Panel B. *With* Unobserved Heterogeneity

	Model 1					
Variables	Prepayment			Default		
	Parameter Estimate	T-ratios		Parameter Estimate	T-ratios	
MEDIUM	-0.1827	-1.4213		0.5318	3.1630	***
LARGE	-0.4243	-2.0689	**	1.0347	5.3937	***
AMZDUMMY	-0.0668	-0.2692		-0.9214	-6.3025	***
ACRDUMMY	-0.7940	-1.2363		0.3827	1.5214	
GPMDUMMY	-0.0414	-0.2209		0.2763	1.9026	*
APARTMENT	0.0032	0.0183		0.1857	0.8883	
INDUSTRIAL	0.1039	0.6120		0.0922	0.4668	
OFFICE	0.0119	0.0671		-0.1129	-0.4681	
INDIVIDUAL	0.1196	0.4993		0.0246	0.0916	
PARTNERSHIP	0.2572	1.3522		0.2302	1.3093	
CORPORATION	-0.0160	-0.0730		-0.1022	-0.4770	
ORIGDSC	-0.6641	-1.4743		0.3035	0.6408	
EN	0.5477	2.4000	**	-0.8025	-3.6594	***
ME	0.0933	0.3691		-0.4315	-1.8559	**
SE	0.3683	1.4641		-0.1616	-0.7530	
SW	0.3321	0.9405		0.3839	1.8087	*
WM	0.4095	1.5177		0.2711	1.3312	
WN	0.7280	2.5123	**	0.1872	0.7388	
WP	0.3277	1.4924		-1.0072	-4.6818	***
LTV RATIO	1.4244	1.3750		-0.9782	-1.6185	*
CAL RATIO	16.6352	5.3024	***	8.1763	3.0950	***
DCR	0.6325	3.9950	***	-0.5971	-2.5001	**
BALLOON	4.0430	20.9821	***	1.7263	9.5468	***
LTV SQUARED	-2.0869	-2.6689	***	0.6698	2.9896	***
CAL SQUARED	-24.3292	-1.8023	*	-55.5056	-3.1191	***
LOC1	2.82E-05	0.8126		3.49E-05	0.5302	
LOC2	2.51E-04	0.8694		2.17E-05	0.5591	
MASS2	0.1169 (t-stat: 1.0965)					
Log Likelihood	-4230.0270					

Note: The model is estimated by maximizing the likelihood function of the competing risk hazard model of prepayment and default. Two 5-th order polynomial functions are estimated as the baseline hazard functions (not reported). *, ** and *** indicate significance at 10%, 5%, and 1% levels respectively. Base variables included in the model are small loans, fixed rate loans, retail properties, and other category for ownership. Number of observations for prepayment and default are 610 and 422, respectively.

Table 4 Maximum Likelihood Estimates for Competing Risks Hazard Model of Prepayment and Default with Default defined as modification, in process of foreclosure or foreclosure.

Panel A. *Without* Unobserved Heterogeneity

Variables	Model 1					
	Prepayment			Default		
	Parameter Estimate	T-ratios		Parameter Estimate	T-ratios	
MEDIUM	-0.1808	-1.3924		0.6909	4.5683	***
LARGE	-0.4103	-2.0018	**	1.0953	6.5881	***
AMZDUMMY	-0.1512	-0.5866		-0.9571	-8.6328	***
ACRDUMMY	-0.4587	-0.7112		0.9943	4.9193	***
GPMDUMMY	0.0806	0.4119		0.5847	5.1056	***
APARTMENT	0.0094	0.0533		0.1796	1.0051	
INDUSTRIAL	0.1290	0.7524		0.0905	0.5185	
OFFICE	0.0401	0.2230		0.0006	0.0028	
INDIVIDUAL	0.1117	0.4658		0.1506	0.6032	
PARTNERSHIP	0.2433	1.2821		0.1836	1.2004	
CORPORATION	-0.0189	-0.0866		0.0620	0.3354	
ORIGDSC	-0.7599	-1.6614	*	-0.0515	-0.1334	
EN	0.5298	2.3264	**	-0.4378	-2.3807	***
ME	0.0862	0.3425		-0.2099	-1.0936	**
SE	0.3836	1.5285		-0.0789	-0.4416	
SW	0.3269	0.9241		0.3409	1.9336	*
WM	0.3940	1.4524		0.4481	2.6983	***
WN	0.6721	2.3187	**	-0.0174	-0.0833	
WP	0.3300	1.5123		-0.8285	-4.5738	***
LTV RATIO	1.2217	1.2162		-0.9361	-2.1438	**
CAL RATIO	16.9060	5.3178	***	8.3328	3.7918	***
DCR	0.6420	4.0145	***	-0.8186	-4.2385	***
BALLOON	4.0243	21.2312	***	1.6322	11.2230	***
LTV SQUARED	-1.8085	-2.4655	**	0.6834	4.1407	***
CAL SQUARED	-24.7184	-1.8268	*	-58.0678	-4.3660	***
LOC1	3.38E-05	0.8256		3.16E-04	1.0313	
LOC2						
MASS2						
Log Likelihood	-4543.2469					

Note: The model is estimated by maximizing the likelihood function of the competing risk hazard model of prepayment and default. Two 5-th order polynomial functions are estimated as the baseline hazard functions (not reported). *, ** and *** indicate significance at 10%, 5%, and 1% levels respectively. Base variables included in the model are small loans, fixed rate loans, retail properties, and other category for ownership. Number of observations for prepayment and default are 599 and 534, respectively.

Table 4 Maximum Likelihood Estimates for Competing Risks Hazard Model of Prepayment and Default with Default defined as modification, in process of foreclosure or foreclosure.

Panel B. *With* Unobserved Heterogeneity

	Model 1					
Variables	Prepayment			Default		
	Parameter Estimate	T-ratios		Parameter Estimate	T-ratios	
MEDIUM	-0.1810	-1.3788		0.6332	3.6657	***
LARGE	-0.4291	-2.0100	**	1.1606	5.7818	***
AMZDUMMY	-0.1315	-0.5072		-1.1768	-7.5880	***
ACRDUMMY	-0.5546	-0.8665		1.6985	5.2336	***
GPMDUMMY	0.0533	0.2644		0.8251	5.3744	***
APARTMENT	0.0065	0.0369		0.1548	0.7229	
INDUSTRIAL	0.1323	0.7663		-0.0600	-0.2945	
OFFICE	0.0412	0.2277		-0.1230	-0.5007	
INDIVIDUAL	0.1165	0.4813		0.1544	0.5333	
PARTNERSHIP	0.2479	1.2963		0.3505	1.7922	*
CORPORATION	-0.0195	-0.0884		0.1458	0.6304	
ORIGDSC	-0.7441	-1.6071		-0.6519	-1.3246	
EN	0.5363	2.3190	**	-0.7437	-3.2605	***
ME	0.0740	0.2895		-0.2901	-1.1679	
SE	0.3798	1.4907		-0.1337	-0.5604	
SW	0.3264	0.9062		0.3519	1.4445	
WM	0.3812	1.3777		0.4036	1.8111	*
WN	0.6847	2.3297	**	-0.0923	-0.3229	
WP	0.3336	1.5056		-1.0882	-4.7363	***
LTV RATIO	1.2348	1.2095		-0.1620	-0.2688	
CAL RATIO	17.0216	5.3527	***	7.3919	2.8278	***
DCR	0.6551	4.1083	***	-0.9085	-3.8316	***
BALLOON	4.0237	20.7738	***	1.5323	8.0287	***
LTV SQUARED	-1.8434	-2.4106	**	0.7651	3.4370	***
CAL SQUARED	-24.6983	-1.8100	*	-71.1861	-4.1460	***
LOC1	2.95E-05	0.7942		4.00E-05	0.8009	
LOC2	5.44E-04	0.9044		4.41E-05	0.8629	
MASS2	0.6702 (t-stat: 4.6641 ***)					
Log Likelihood	-4532.1745					

Note: The model is estimated by maximizing the likelihood function of the competing risk hazard model of prepayment and default. Two 5-th order polynomial functions are estimated as the baseline hazard functions (not reported). *, ** and *** indicate significance at 10%, 5%, and 1% levels respectively. Base variables included in the model are small loans, fixed rate loans, retail properties, and other category for ownership. Number of observations for prepayment and default are 599 and 534, respectively.

Table 5. Maximum Likelihood Estimates for Competing Risks Hazard Model of Prepayment and Default with default defined as 90 days delinquency.

Panel A. *Without* Unobserved Heterogeneity

Variables	Model 1					
	Prepayment			Default		
	Parameter Estimate	T-ratios		Parameter Estimate	T-ratios	
MEDIUM	-0.2210	-1.6942	*	0.6386	4.6352	***
LARGE	-0.4062	-1.9759	**	0.9864	6.4098	***
AMZDUMMY	-0.1773	-0.6871		-0.9150	-8.4474	***
ACRDUMMY	-0.4886	-0.7499		0.9475	4.5986	***
GPMDUMMY	0.1184	0.6019		0.6603	5.9749	***
APARTMENT	0.0020	0.0111		0.2117	1.2440	
INDUSTRIAL	0.1221	0.7124		0.2087	1.2884	
OFFICE	0.0469	0.2604		-0.0009	-0.0045	
INDIVIDUAL	0.1703	0.7108		0.0200	0.0874	
PARTNERSHIP	0.2887	1.5205		0.0520	0.3449	
CORPORATION	0.0262	0.1204		-0.0482	-0.2743	
ORIGDSC	-0.8375	-1.7743	*	-0.1226	-0.3394	
EN	0.5272	2.3167	**	-0.4163	-2.3533	**
ME	0.0971	0.3859		-0.1311	-0.7088	
SE	0.3947	1.5731		0.1079	0.6428	
SW	0.3185	0.9031		0.3575	2.0599	**
WM	0.4007	1.4794		0.4459	2.7133	**
WN	0.6879	2.3497	**	-0.0264	-0.1297	
WP	0.3105	1.4243		-0.6983	-4.0045	***
LTV RATIO	0.7955	0.7888		-0.8402	-2.1073	**
CAL RATIO	17.3108	5.4098	***	7.4147	4.1787	***
DCR	0.6394	3.9033	***	-0.6401	-3.4646	***
BALLOON	3.9959	20.8428	***	1.7835	13.0687	***
LTV SQUARED	-1.5353	-2.0747	**	0.6497	4.3322	***
CAL SQUARED	-26.0255	-1.9124	*	-55.1681	-4.7262	***
LOC1	4.81E-05	0.8272		6.08E-04	1.1700	
LOC2						
MASS2						
Log Likelihood	-4752.0834					

Note: The model is estimated by maximizing the likelihood function of the competing risk hazard model of prepayment and default. Two 5-th order polynomial functions are estimated as the baseline hazard functions (not reported). *, ** and *** indicate significance at 10%, 5%, and 1% levels respectively. Base variables included in the model are small loans, fixed rate loans, retail properties, and other category for ownership. Number of observations for prepayment and default are 576 and 596, respectively.

Table 5. Maximum Likelihood Estimates for Competing Risks Hazard Model of Prepayment and Default with default defined as 90 days delinquency.

Panel B. *With* Unobserved Heterogeneity

	Model 1					
Variables	Prepayment			Default		
	Parameter Estimate	T-ratios		Parameter Estimate	T-ratios	
MEDIUM	-0.2211	-1.6573	*	0.6388	4.0380	***
LARGE	-0.4392	-2.0522	**	1.0647	5.7203	***
AMZDUMMY	-0.1544	-0.5889		-1.0700	-7.1747	***
ACRDUMMY	-0.6416	-0.9996		1.6365	4.8333	***
GPMDUMMY	0.0760	0.3718		0.8354	5.6527	***
APARTMENT	-0.0094	-0.0523		0.1987	0.9803	
INDUSTRIAL	0.1171	0.6708		0.1496	0.7913	
OFFICE	0.0411	0.2248		-0.0684	-0.2955	
INDIVIDUAL	0.1699	0.6935		0.0875	0.3276	
PARTNERSHIP	0.2974	1.5336		0.2050	1.1037	
CORPORATION	0.0227	0.1019		0.0807	0.3753	
ORIGDSC	-0.8093	-1.6582	*	-0.7060	-1.5211	
EN	0.5447	2.3262	**	-0.6419	-2.9351	**
ME	0.0860	0.3315		-0.1889	-0.7990	
SE	0.4032	1.5579		0.0961	0.4322	
SW	0.3190	0.8748		0.4436	1.8657	*
WM	0.3862	1.3830		0.3956	1.8255	*
WN	0.7123	2.3657	**	-0.0994	-0.3594	
WP	0.3235	1.4419		-0.9133	-4.2064	***
LTV RATIO	0.8257	0.8023		0.0636	0.1090	
CAL RATIO	17.6215	5.4808	***	6.0706	3.0229	***
DCR	0.6689	4.0860	***	-0.7137	-2.9640	***
BALLOON	3.9968	19.8841	***	1.7105	9.6520	***
LTV SQUARED	-1.6044	-2.0757	**	0.7137	3.3754	***
CAL SQUARED	-26.0009	-1.8763	*	-66.8017	-4.5438	***
LOC1	3.7765E-05	0.7901		6.1691E-05	0.8049	
LOC2	8.1671E-04	0.9846		6.5893E-05	0.9301	
MASS2	0.6445 (t-stat: 4.9459 ***)					
Log Likelihood	-4740.4392					

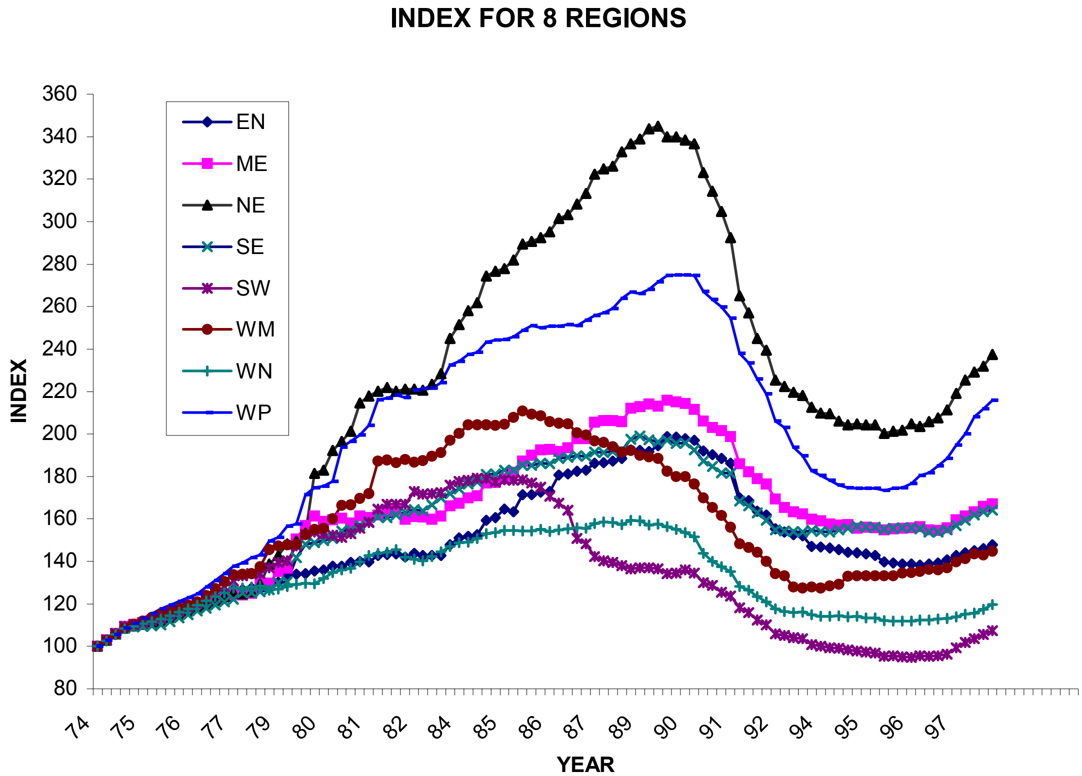
Note: The model is estimated by maximizing the likelihood function of the competing risk hazard model of prepayment and default. Two 5-th order polynomial functions are estimated as the baseline hazard functions (not reported). *, ** and *** indicate significance at 10%, 5%, and 1% levels respectively. Base variables included in the model are small loans, fixed rate loans, retail properties, and other category for ownership. Number of observations for prepayment and default are 576 and 596, respectively.

Table 6. Cox Partial Likelihood Estimates for the Independent Risks of Mortgage Prepayment and Default

	Model 3					
Variables	Prepayment			Default		
	Parameter Estimate	T-ratios		Parameter Estimate	T-ratios	
MEDIUM	-0.232	-2.078	**	0.638	4.438	***
LARGE	-0.643	-3.841	***	0.981	6.324	***
AMZDUMMY	-0.067	-0.363		-0.915	-8.069	***
ACRDUMMY	-1.005	-1.524		1.161	6.184	***
GPMDUMMY	-0.054	-0.334		0.549	4.675	***
APARTMENT	-0.184	-1.207		0.005	0.028	
INDUSTRIAL	-0.100	-0.729		0.066	0.436	
OFFICE	-0.118	-0.798		-0.059	-0.321	
INDIVIDUAL	0.008	0.045		0.063	0.275	
PARTNERSHIP	0.218	1.406		0.090	0.606	
CORPORATION	-0.200	-1.131		0.021	0.118	
ORIGDSC	-1.025	-2.986	**	0.247	0.779	
LTV RATIO	3.641	2.868	**	-1.325	-3.235	***
CAL RATIO	23.621	10.190	***	8.509	5.406	***
DCR	1.229	7.657	***	-1.527	-6.665	***
BALLOON	4.963	33.210	***	1.703	11.791	***
LTV SQUARED	-3.121	-3.690	***	0.801	5.752	***
CAL SQUARED	-53.040	-4.572	***	-64.860	-6.062	***
LOC1						
LOC2						
MASS2						
Partial Log Likelihood	-1682.034			-2258.405		

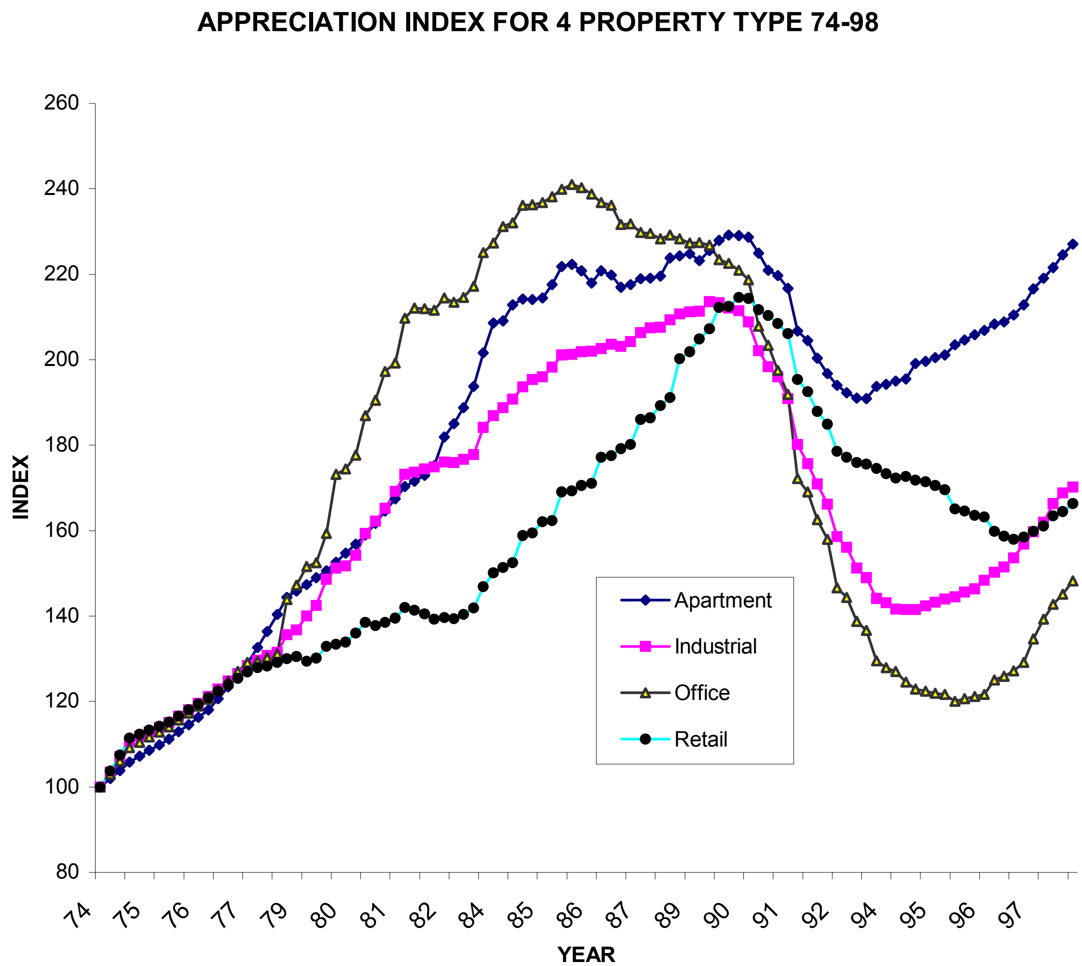
Note: The model is estimated with Cox's Partial Likelihood (CPL) approach, which does not require specification of baseline hazard functions. Prepayment and default functions are estimated separately. In the estimation for one risk, termination events by the other risk are taken as censored. The value of the Partial Log Likelihood is not comparable to the Full Log Likelihood Value one in the competing risk specification of table 4 and 5. *, ** and *** indicate significance at 10%, 5%, and 1% levels respectively.

Figure 1. NCREIF Property Value Appreciation Index by Region



Notes: The property index is calculated from NCREIF appreciation returns from 1978 to 1998. Marshall and Swift construction cost figures are used as a substitute for years 1974-1977, when the NCREIF data were not available.

Figure 2. NCREIF Property Value Appreciation Index by Property Type



Notes: The property index is calculated from NCREIF appreciation returns from 1978 to 1998. Marshall and Swift construction cost figures are used as a substitute for years 1974-1977, when the NCREIF data were not available.

Figure 3. ACLI Mortgage Commitment Rate vs. Mortgage Coupon Rate in the Sample

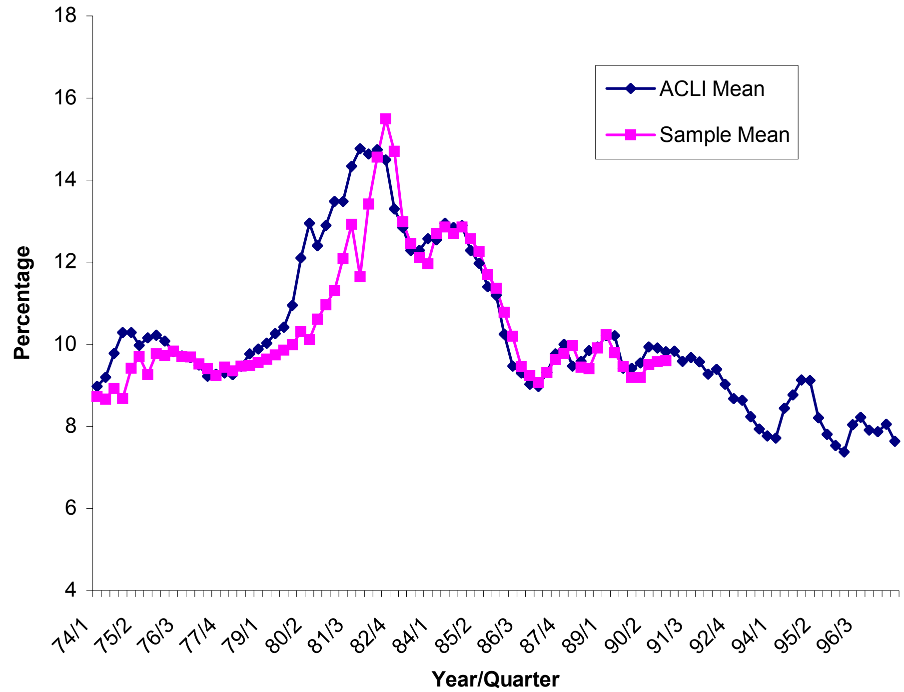
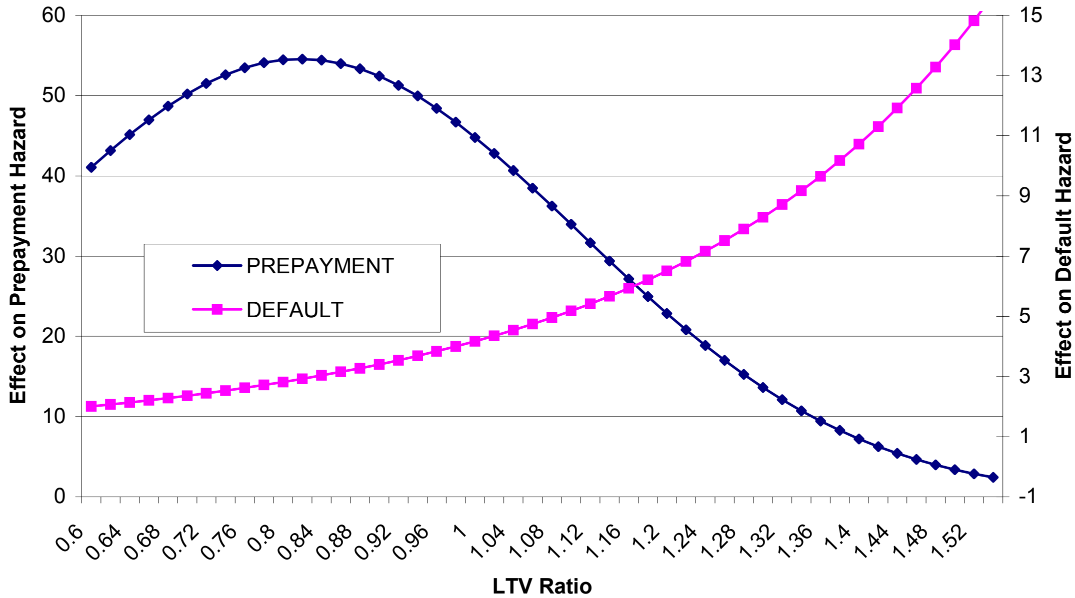
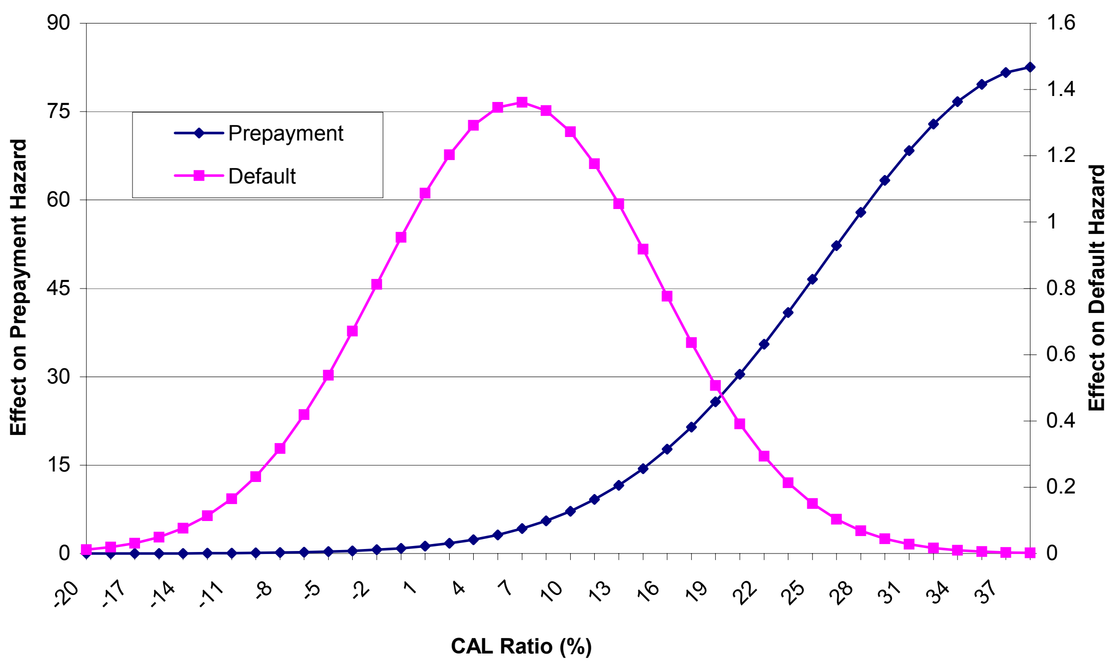


Figure 4. Effects of Call and Put on Prepayment and Default Hazard

4A. The Effect of LTV Ration on Prepayment and Default Hazard

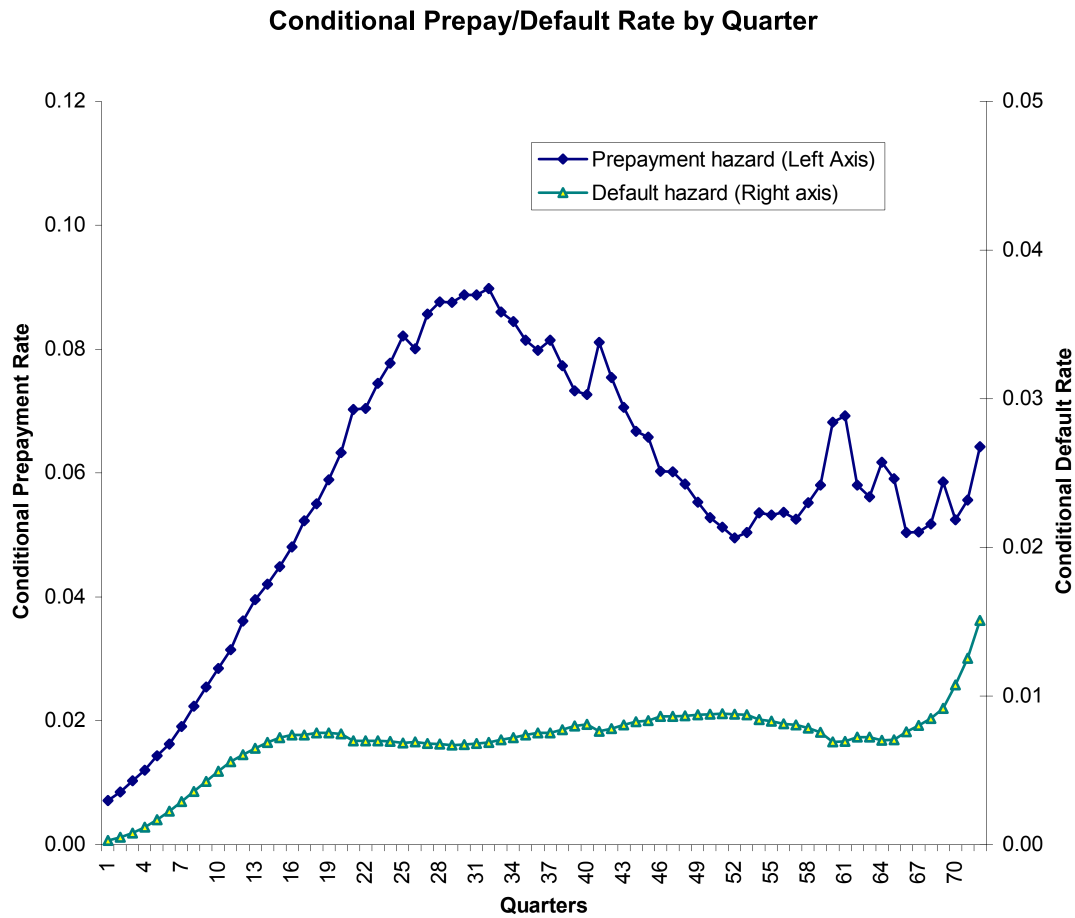


4B. Effect of Call Ratio on Prepayment and Default Hazard



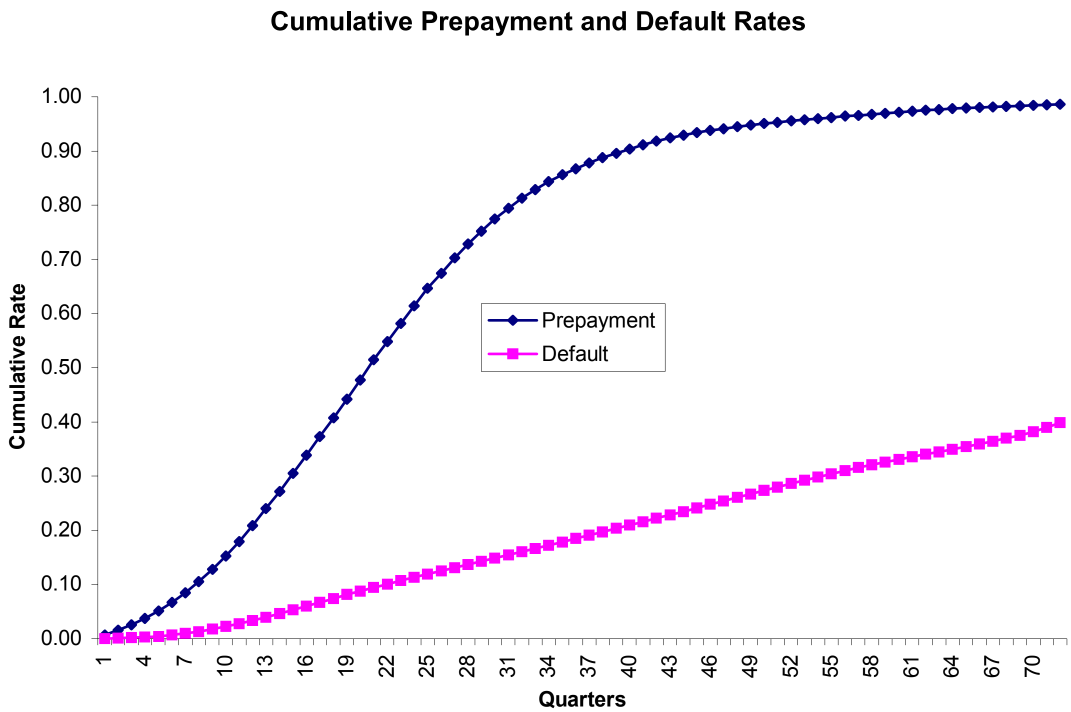
Note: Calculated based on coefficients estimated in Table 3, Panel A. Effect on hazard rates equal to $exp(xb_1+x^2b_2)$, where b_1 and b_2 are coefficients for the linear and squared terms, respectively.

Figure 5. Estimated Conditional Quarterly Default and Prepayment



Note: Baseline function is estimated by fitting 5-th Order Polynomial Functions. Median sample values of LTV and DCR ratios are used to compute the predicted conditional default and prepayment rates. CAL RATIO is set to 0.

Figure 6. Estimated Cumulative Prepayment and Default Rate



Note: Baseline function is estimated by fitting 5-th Order Polynomial Functions. Median sample values of LTV and DCR ratios are used to compute the predicted conditional default and prepayment rates. CAL RATIO is set to 0.